Question

The mean and the standard deviation of 20 items is found to be 10 and 2 respectively. At the time of checking it was found that one item with value 8 was incorrect. Calculate the mean and standard deviation if the wrong item is omitted.

Solution

1) Let's first find the correct mean. By the definition of the mean we have:

$$\bar{x} = \frac{\sum x_i}{n},$$

where, $\bar{x}$ is the mean, $\sum x_i$ is the sum of the items, n is the number of the items.

Substituting the mean and the number of the items into the formula we can calculate the sum of the items:

$$\sum x_i = \bar{x}n = 10 \cdot 20 = 200.$$

Because one item with value 8 was incorrect and it is omitted we must subtract this item from the sum of the items:

$$\sum(x_i)_C = 200 - 8 = 192.$$

Thus, we have the number of items $n = 19$ and correct sum of the items $\sum x_i = 192$ and can calculate the correct mean:

$$\bar{x}_C = \frac{\sum(x_i)_C}{n} = \frac{192}{19} = 10.1.$$

2) Let's find the correct standard deviation. From the condition of the question we know that the standard deviation $\sigma = 2$. Then the variance σ^2 will be:

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = 2^2 = 4,$$

$$\frac{\sum x_i^2}{20} - 10^2 = 4,$$

$$\sum x_i^2 = (4 + 100) \cdot 20 = 2080.$$

Because wrong item 8 is omitted we get:

$$\Sigma\left(x_i^2\right)_C = 2080 - 8^2 = 2080 - 64 = 2016.$$

Therefore, we can calculate the correct standard deviation:

$$\sigma_C = \sqrt{\frac{\Sigma\left(x_i^2\right)_C}{n} - \left(\frac{\Sigma(x_i)_C}{n}\right)^2} = \sqrt{\frac{2016}{19} - \left(\frac{192}{19}\right)^2} = \sqrt{\frac{2016 \cdot 19 - 192^2}{19^2}} = \frac{\sqrt{1440}}{19} = 1.99.$$

Answer:

1) $\bar{x}_C = 10.1$.

2) $\sigma_C = 1.99$.