A bag contains 10 white and 3 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. Find the probability that this procedure come to an end at the 6th draw.

Solution

If the procedure for drawing balls has come to an end at the 6th draw, all but one black ball must be drawn in the first 5 draws.

The required event is we drawn 2 black balls and 3 white balls in the first 5 draws AND the last third black ball was drawn at the 6th draw.

The probability of the last third black ball was drawn at the 6th draw is

$$\frac{1}{8}.$$

It is because we have 8 balls and only one of them is black at the 6th draw.

Let number of possible ways to select is NOPWTS.

The probability of drawing 2 black balls and 3 white balls in the first 5 draws is

$$\frac{\text{NOPWTS } \mathbf{2} \text{ } black \text{ } balls \text{ } from \text{ } \mathbf{3} \cdot \text{NOPWTS } \mathbf{3} \text{ } white \text{ } balls \text{ } from \text{ } \mathbf{10}}{\text{NOPWTS } \mathbf{5} \text{ } balls \text{ } from \text{ } \mathbf{13}} = \frac{\binom{3}{2}\binom{10}{6-3}}{\binom{10+3}{6-1}}$$

The probability of the required event is

$$p = \frac{\binom{3}{2}\binom{10}{6-3}}{\binom{10+3}{6-1}}\frac{1}{8} = 3 \cdot \frac{10!}{3! \, 7!} \cdot \frac{5! \, 8!}{13!}\frac{1}{8} = 3 \cdot \frac{5 \cdot 4}{13 \cdot 12 \cdot 11} \approx 0.0350.$$

Answer: 0.0350.