What is the largest rectangular area one can enclose with 38 inches of string?
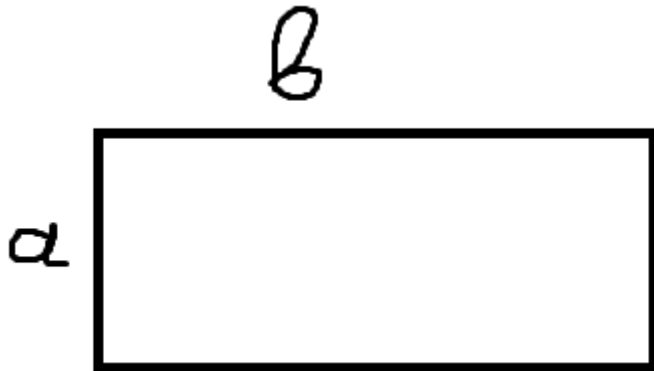
Solution

Let a and b be the lengths of sides of a rectangular.

The perimeter of the rectangular equals 2a+2b=38, hence b=19 $-$ a.

Area of the rectangle is given by

$$S = ab = a(19 - a) = 19a - a^2, \qquad (1)$$

where $S = S(a)$ is the function of a.

To find the maximum of area, we need to solve the equation

$$S' = 0,$$

where S' is the derivative of S with respect to a.

$$S' = (19a - a^2)' = 19 - 2a$$

Solution of $19 - 2a = 0$ is $a = \dfrac{19}{2} = 9.5$

Substituting $a = 9.5$ into expression (1) gives

$S = a(19 - a) = 9.5 \cdot (19 - 9.5) = 9.5 \cdot 9.5 = 90.25$ (in^2).

Answer: 90.25 in^2.