

Answer on Question #54828 – Math – Algebra

1. Solve the following quadratic equation for the indicated variable. (Enter your answers as a comma-separated list.)

$$y^2 - 8y = x^2 - 16$$

for x

x =

2. Solve the following quadratic equation for the indicated variable. (Enter your answers as a comma-separated list.)

$$y^2 - 5y = 5x$$

for y

y =

Solution

1. Transform equation

$$y^2 - 8y = x^2 - 16 \quad (1)$$

$$y^2 - 8y + 16 = x^2$$

$$(y - 4)^2 = x^2.$$

Hence

$$x = y - 4 \text{ or } x = 4 - y. \quad (2)$$

Both solutions (2) exist for any values y.

The quadratic equation (1) has two solutions (2) if $y \neq 4$ and it has one solution $x = 0$ if $y = 4$.

2. Transform equation

$$y^2 - 5y = 5x \rightarrow$$

$$y^2 - 5y - 5x = 0. \quad (3)$$

We find the discriminant $D = 25 + 4 \times 1 \times 5x = 25 + 20x$.

Solutions of the equation (3) will be $y_1 = \frac{5 - \sqrt{25 + 20x}}{2}$, $y_2 = \frac{5 + \sqrt{25 + 20x}}{2}$.

Inequality $25 + 20x > 0$ is equivalent to $x > -\frac{25}{20}$, $x > -1.25$, that is, $x \in (-1.25; +\infty)$.

Inequality $25 + 20x < 0$ is equivalent to $x < -\frac{25}{20}$, $x < -1.25$, that is, $x \in (-\infty; -1.25)$.

As a result, equation (3) has two solutions $y_1 = \frac{5 - \sqrt{25 + 20x}}{2}$, $y_2 = \frac{5 + \sqrt{25 + 20x}}{2}$ if $x > -1.25$.

Equation (3) has one solution $y = \frac{5}{2}$ if $x = -1.25$.

Equation (3) has no solutions for real y if $x < -1.25$.

Answer: 1) two solutions $x_1 = y - 4$, $x_2 = 4 - y$ if $y \neq 4$;

one solution $x = 0$ if $y = 4$.

2) two solutions $y_1 = \frac{5 - \sqrt{25 + 20x}}{2}$, $y_2 = \frac{5 + \sqrt{25 + 20x}}{2}$ if $x > -1.25$;

one solution $y = \frac{5}{2}$ if $x = -1.25$;

no solutions y if $x < -1.25$.