Answer on Question #54820 – Math – Real Analysis

Question 1. f is differentiable at c, prove that:

1) $f'(c)=\lim\limits_{h\to 0}\dfrac{f(c+h)-f(c)}{h}$

2) $f'(c)=\lim\limits_{h\to 0}\dfrac{f(c+h)-f(c-h)}{2h}$

Solution

1) Since f is differentiable at c, we may write $f(c+h)=f(c)+f'(c)h+\bar{o}(h)$. Thus we obtain

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}=\lim_{h\to 0}\frac{f(c)+f'(c)h+\bar{o}(h)-f(c)}{h}=f'(c)+\lim_{h\to 0}\frac{\bar{o}(h)}{h}=f'(c)+0=f'(c)$$

2) Since f is differentiable at c, we may write also $f(c-h)=f(c)+f'(c)(-h)+\bar{o}(h)$. Thus we obtain

$$\lim_{h\to 0}\frac{f(c+h)-f(c-h)}{2h}=\lim_{h\to 0}\frac{f(c)+f'(c)h+\bar{o}(h)-(f(c)-f'(c)h+\bar{o}(h))}{2h}=f'(c)+\lim_{h\to 0}\frac{\bar{o}(h)}{h}=f'(c)+0=f'(c)$$

Question 2. Let f:R→R .The function f is even if f(−x)=f(x) for all $x\in R$, and odd if f(−x)=−f(x) for all $x\in R$ if f is differentiable. Prove that **1)** $f'(x)$ is odd when f is even, and **2)** $f'(x)$ is even when f is odd.

Solution

1) Let f be even. This means that $f(-x)=f(x)$.
Denote $(-x)=t$ and rewrite

$$f(t)=f(-t). \qquad \textbf{(1)}$$

Differentiate the previous equality (1) by *t*:

$f'(t)=\left(f(-t)\right)'=f'(-t)\cdot(-t)'=-f'(-t)$.

We got

$f'(t)=-f'(-t)$, or

$$f'(-x)=-f'(x). \qquad \textbf{(2)}$$

The last expression (2) means that $f'(x)$ is odd by the definition.

2) Let f be odd. This means that $f(-x)=-f(x)$.
Denote $(-x)=t$ and rewrite

$$f(t)=-f(-t). \qquad \textbf{(3)}$$

Differentiate the previous equality (3) by *t*:

$f'(t)=\left(-f(-t)\right)'=-f'(-t)\cdot(-t)'=f'(-t)$.

We got

$f'(t)=f'(-t)$, or

$$f'(-x)=f'(x). \qquad \textbf{(4)}$$

The last expression (4) means that $f'(x)$ is even by the definition.