Answer on Question #54791 – Math – Abstract Algebra

1. Given a set $X = \{a, b, c\}$, and a function $\Psi: X \to X$ defined by

$\Psi(a) = b, \Psi(\mathrm{b}) = \mathrm{a}, \Psi(\mathrm{c}) = \mathrm{c}$. The function is

(a)Only onto

(b) only injective

(c) bijective

(d)no solution

2. A matrix $X = \binom{3\ 1}{5\ 2}$ define a function from $R^2 \to R^2$ by $f(X)\binom{a}{b} = (3a + b, 5a + 2b)$. Find the inverse function of f(X)

(a)f −1 X (a,b)=(3a−b,−5a+3b)

(b)f 1 X (a,b)=(2a−b,−5a+3b)

(c)f 5 X (a,b)=(2a−b,−5a+3b)

(d) f 1 X (a,b)=(2a−b,−5a+4b)

3. Which of the following is divisible by 17 for all positive integer n

(a) 7 n +2

(b) 6 n +2

(c) 2.7 n +3.5 n −5

(d)3.5 2n+1 +2 3n+1

Solution

1. I used the definition of bijection. In mathematics, a bijection, bijective function or one-to-one correspondence is a function between the elements of two sets, where every element of one set is paired with exactly one element of the other set, and every element of the other set is paired with exactly one element of the first set. There are no unpaired elements. In mathematical terms, a bijective function *f*: *X* → *Y* is a one-to-one (injective) and onto (surjective) mapping of a set *X* to a set *Y*. In set X every element (a,b or c) is paired with exactly one element (b,a,c) respectively.
2. Given $X = \binom{3\ 1}{5\ 2}$, its determinant is $\det(X) = 3 \cdot 2 - 5 \cdot 1 = 1$. We find the inverse matrix by the major minor rule:
$$X^{-1} = \frac{1}{\det(X)}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}^T = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \Rightarrow X^{-1}\binom{a}{b} = (2a - b, -5a + 3b)$$
3. I substitute n=1 to each choice:
a) 7 n +2 =9 is not divisible by 17

b) $6n+2=8$ is not divisible by 17

c) $2.7n+3.5n-5=2.7+3.5-5=1.2$ is not divisible by 17

d) The condition is written incorrectly or incompletely.

Answer:

1 c

2 b

3 there is no correct answer.