Answer on Question #54790 – Math – Abstract Algebra

1. If R (the set of real number) be the universal set and sets V={yϵR:0< y ≤3} and W={yϵR:2 ≤ y <4 } What is V \W?

2. For sets A and B , if A and B are subset of Z (the set of Integer) which of the following relations between the two subset is true?

(a) $A \cup B = A$

(b)$(A\backslash B) \cap (B\backslash A) = \emptyset$

(c) $(A\backslash B) \cap (B\backslash A) = Z$

(d) $(A\backslash B) \cup (B\backslash A) = \emptyset$

3. Which of the following pair of functions has $f \circ g = g \circ f$

(a)$f(y) = y - 3 \; and \; g(y) = y \cdot \sqrt{3}$

(b) $f(y) = y^{\{5\}} \; and \; g(y) = 3y + 7$

(c) $f(y) = y - 2 \; and \; g(y) = y + 7$

(d) $f(y) = y - 2 \; and \; g(y) = 3y + 7$

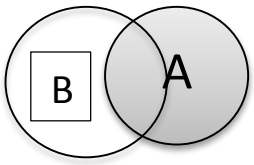
Solution

1. V\W={yϵR:0< y <2} is all elements from V, which do not belong to W.

2. $A \cup B$ means that we have objects from A or B(and they don`t equal each other). That`s why $A \cup B$ is not always equal to A.

$(A\backslash B) \cup (B\backslash A)$. We don`t know whether $A \subset B$ or $B \subset A$ . There are at least two elements: one from *A*, and the other from *B*. That`s why $(A\backslash B) \cup (B\backslash A)$ is not empty.

$(A\backslash B) \cap (B\backslash A)$



We can see from the diagram that this intersection is empty.

It is not equal to Z.

The true relation is (b).

3. $f \circ g = f\left(y \cdot \sqrt{3}\right) = y\sqrt{3} - 3$ and $g \circ f = g(y - 3) = (y - 3)\sqrt{3} = y\sqrt{3} - 3\sqrt{3} \Rightarrow$

$$f \circ g \neq g \circ f$$

$f \circ g = f(3y + 7\,) = (3y + 7\,)^{\{5\}}$ and $g \circ f = g\left(y^{\{5\}}\right) = 3y^{\{5\}} + 7 \Rightarrow f \circ g \neq g \circ f$

$f \circ g = f(y + 7\,) = y + 7 - 2 = y + 5$ and $g \circ f = g(y - 2) = y - 2 + 7 = y + 5 \Rightarrow$

$$f \circ g = g \circ f$$

$f \circ g = f(3y + 7) = 3y + 7 - 2 = 3y + 5$ and

$g \circ f = g(y - 2) = 3(y - 2) + 7 = 3y + 1 \Rightarrow f \circ g \neq g \circ f$

Answer: 1. V\W={yϵR: 0< y <2}

2. (b)

3. (c)