Answer on Question#54769 –Math– Calculus

For each of the following functions, find the first three derivatives (without simplifying) then generalize the pattern to find a formula for the nth derivative of f. Evaluate the nth derivative for the value of x given.

a) f(x)=x^(2/3), x=1

b) f(x)=ln(1+2x), x=0

Solution.

Let us find the derivative using the basic rules of differentiation.

a) The derivative of the function of a function *h*(*x*) = *f*(*g*(*x*)) with respect to *x* is $h'(x) = f'(x) \times g'(x)$. Using the table of derivatives we will find a derivative power of x. $(x^n)' = n \times x^{n-1}$). We get three derivatives

$$f'(x) = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}};$$

$$f''(x) = \frac{2}{3}\left(-\frac{2}{3}\right)x^{-\frac{1}{3}-1} = \frac{2}{3}\left(-\frac{1}{3}\right)x^{-\frac{4}{3}};$$

$$f'''(x) = \frac{2}{3}\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)x^{-\frac{4}{3}-1} = \frac{2}{3}\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)x^{-\frac{7}{3}};$$

Examining generalize the pattern we get

$$f^n(x) = \frac{2}{3}\left(\frac{2}{3}-1\right)\dots\left(\frac{2}{3}-(n-1)\right)x^{\frac{2}{3}-n}.$$

Value of the nth derivative function f for x=1 is equal

$$f^n(1) = \frac{2}{3}\left(\frac{2}{3}-1\right)\dots\left(\frac{2}{3}-(n-1)\right).$$

b) The derivative of the function of a function *h*(*x*) = *f*(*g*(*x*)) with respect to *x* is $h'(x) = f'(x) \times g'(x)$. Using the table of derivatives we will find derivative logarithmic functions and derivative power of x $(\ln(x))' = \frac{1}{x}$ $(x^n)' = n \times x^{n-1}$.

$$f'(x) = \frac{1}{1+2x}(1+2x)' = \frac{2}{1+2x};$$

$$f''(x) = 2((1+2x)^{-1})' = 2\left(-\frac{2}{(1+2x)^2}\right)(1+2x)' = -\frac{4}{(1+2x)^2};$$

$$f'''(x) = -4((1+2x)^{-2})' = -4\left(-\frac{2}{(1+2x)^3}\right)(1+2x)' = \frac{16}{(1+2x)^3};$$

Examining generalise the pattern we get

$$f^n(x) = \frac{(-1)^{n+1} \times 2^n \times (n-1)!}{(1+2x)^n}.$$

Value of the nth derivative function f for x=0 is equal

$f^n(0) = (-1)^{n+1} \times 2^n \times (n-1)!$.

Answer: **a)** $f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$; $f''(x) = \frac{2}{3}\left(-\frac{1}{3}\right)x^{-\frac{4}{3}}$; $f'''(x) = \frac{2}{3}\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)x^{-\frac{7}{3}}$; $f^n(x) = \frac{2}{3}\left(\frac{2}{3}-1\right)\dots\left(\frac{2}{3}-(n-1)\right)x^{\frac{2}{3}-n}$; $f^n(1) = \frac{2}{3}\left(\frac{2}{3}-1\right)\dots\left(\frac{2}{3}-(n-1)\right)$. b) $f'(x) = \frac{2}{1+2x}$; $f''(x) = -\frac{4}{(1+2x)^2}$; $f'''(x) = \frac{16}{(1+2x)^3}$; $f^n(x) = \frac{(-1)^{n+1}\times 2^n \times (n-1)!}{(1+2x)^n}$; $f^n(0) = (-1)^{n+1} \times 2^n \times (n-1)!$.