Answer on Question# 54767 – Math / Calculus

Question: Find the values of a and b that make f continuous for $x \in \mathbb{R}$

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & \text{if } x < 2; \\ ax^2 - bx + 3, & if\ 2 \le x < 3; \\ 2x - a + b, & if\ x \ge 3 \end{cases}$$

Solution:

Consider two points $x_1 = 2$ and $x_2 = 3$ and write the conditions of continuity at these points

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) \qquad \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$$

Evaluate each limit

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^-} (x+2) = 4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (ax^2 - bx + 3) = 4a - 2b + 3$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2x - a + b) = 6 - a + b$$

If f(x) is continuous, first limit equals to second one, and third limit equals to fourth. We obtain system of two equations

$$\begin{cases} 4a - 2b + 3 = 4 \\ 9a - 3b + 3 = 6 - a + b \end{cases}$$

Solve it

$$\begin{cases} 4a - 2b = 1 \\ 10a - 4b = 3 \end{cases}$$

$$\begin{cases} 8a - 4b = 2 \\ 10a - 4b = 3 \end{cases}$$

Subtract first equation from second: $2a = 1$ then $a = \frac{1}{2}$

Substitute into first equation and obtain 2-2b=1 then $b = \frac{1}{2}$.

For continuity check values at two points $f(x_1) = f(2) = \frac{1}{2} \cdot 4 - \frac{1}{2} \cdot 2 + 3 = 4$

$$f(x_2) = f(3) = 6 - \frac{1}{2} + \frac{1}{2} = 6$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax^2 - bx + 3) = \frac{1}{2} \cdot 9 - \frac{1}{2} \cdot 3 + 3 = 6$$

Answer: $a = b = \frac{1}{2}$