Answer on Question #54766 - Math - Calculus

a) $\lim\limits_{n\to+\infty}\dfrac{n^3-1}{2n^3+1}=\lim\limits_{n\to+\infty}\dfrac{\not{n}^3(1-\frac{1}{n^3})}{\not{n}^3(2+\frac{1}{n^3})}=\lim\limits_{n\to+\infty}\dfrac{1-\frac{1}{n^3}}{2+\frac{1}{n^3}}$. Using the property $\lim\limits_{n\to+\infty}\dfrac{a_n}{b_n}=\dfrac{\lim\limits_{n\to+\infty}a_n}{\lim\limits_{n\to+\infty}b_n}$ if $\lim\limits_{n\to+\infty}a_n$, $\lim\limits_{n\to+\infty}b_n$ exist and $\lim\limits_{n\to+\infty}b_n\neq 0$ we have $\lim\limits_{n\to+\infty}\dfrac{1-\frac{1}{n^3}}{2+\frac{1}{n^3}}=$
$=\dfrac{\lim\limits_{n\to+\infty}(1-\frac{1}{n^3})}{\lim\limits_{n\to+\infty}(2+\frac{1}{n^3})}=\dfrac{1-\lim\limits_{n\to+\infty}\frac{1}{n^3}}{2+\lim\limits_{n\to+\infty}\frac{1}{n^3}}$. Since $\lim\limits_{n\to+\infty}\dfrac{1}{n^3}=0$ we have $\dfrac{1-\lim\limits_{n\to+\infty}\frac{1}{n^3}}{2+\lim\limits_{n\to+\infty}\frac{1}{n^3}}=\dfrac{1-0}{2+0}=$ $\dfrac{1}{2}=0.5$.

b) $\lim\limits_{n\to+\infty}\dfrac{\ln n}{\ln 2n}=\lim\limits_{n\to+\infty}\dfrac{\ln n}{\ln 2+\ln n}=\lim\limits_{n\to+\infty}\dfrac{\not{\ln} n}{\not{\ln} n(1+\frac{\ln 2}{\ln n})}=\lim\limits_{n\to+\infty}\dfrac{1}{1+\frac{\ln 2}{\ln n}}$. Using the property $\lim\limits_{n\to+\infty}\dfrac{a_n}{b_n}=\dfrac{\lim\limits_{n\to+\infty}a_n}{\lim\limits_{n\to+\infty}b_n}$ if $\lim\limits_{n\to+\infty}a_n$, $\lim\limits_{n\to+\infty}b_n$ exist and $\lim\limits_{n\to+\infty}b_n\neq 0$ we have $\lim\limits_{n\to+\infty}\dfrac{1}{1+\frac{\ln 2}{\ln n}}=\dfrac{\lim\limits_{n\to+\infty}1}{\lim\limits_{n\to+\infty}(1+\frac{\ln 2}{\ln n})}=\dfrac{1}{1+\lim\limits_{n\to+\infty}\frac{\ln 2}{\ln n}}=\dfrac{1}{1+\ln 2\lim\limits_{n\to+\infty}\frac{1}{\ln n}}$.
Since $\lim\limits_{n\to+\infty}\dfrac{1}{\ln n}=0$ we have $\dfrac{1}{1+\ln 2\lim\limits_{n\to+\infty}\frac{1}{\ln n}}=\dfrac{1}{1+\ln 2\cdot 0}=1$.

c) $\lim\limits_{n\to+\infty}\left(1+\dfrac{2}{n}\right)^n=\lim\limits_{n\to+\infty}\left(\left(1+\dfrac{2}{n}\right)^{\frac{n}{2}}\right)^2$. If n is large then $\alpha=\dfrac{2}{n}$ is very small. Let us $\alpha=\dfrac{2}{n}$ then $\lim\limits_{n\to+\infty}\left(\left(1+\dfrac{2}{n}\right)^{\frac{n}{2}}\right)^2=\lim\limits_{\alpha\to 0}\left((1+\alpha)^{\frac{1}{\alpha}}\right)^2$. Since $\lim\limits_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e$ we have $\lim\limits_{\alpha\to 0}\left((1+\alpha)^{\frac{1}{\alpha}}\right)^2=e^2$.