Answer on Question #54754 - Math - Algebra

Solve the following equations:

1) $49^{3-2x} = \sqrt{243}$

2) $1331^{2x} = \frac{1}{121^{5-x}}$

3) $\left(\frac{5}{3}\right)^{x+2} = \left(\frac{27}{125}\right)^{2x+3}$

Solution

1) $49^{3-2x} = \sqrt{243}$

$7^{2(3-2x)} = 3^{\frac{5}{2}}$

$2(3-2x) = \log_7\left(3^{\frac{5}{2}}\right)$

$6-4x = \log_7\left(3^{\frac{5}{2}}\right)$

$6-4x = \frac{5}{2}\log_7(3)$

$x = \frac{\frac{5}{2}\log_7(3)-6}{-4}$

Answer: $x = \frac{3}{2} - \frac{5}{8}\log_7(3)$.

Check: $49^{3-2\left(\frac{3}{2}-\frac{5}{8}\log_7(3)\right)} = 49^{3-3+\frac{5}{4}\log_7 3} = 7^{2\left(\frac{5}{4}\log_7 3\right)} = 7^{\log_7 3^{\frac{5}{2}}} = 3^{\frac{5}{2}} = \sqrt{3^5} = \sqrt{243}$.

2) $1331^{2x} = \frac{1}{121^{5-x}}$

$11^{3(2x)} = \frac{1}{11^{2(5-x)}}$

$11^{6x} = 11^{-2(5-x)}$

$6x = -2(5-x)$

$6x + 10 - 2x = 0$

Answer: $x = -\frac{5}{2}$.

Check: $1331^{2\left(-\frac{5}{2}\right)} = 11^{3(-5)} = 11^{-15} = \frac{1}{11^{15}} = \frac{1}{11^{2\left(\frac{15}{2}\right)}} = \frac{1}{121^{\frac{15}{2}}} = \frac{1}{121^{5-\left(-\frac{5}{2}\right)}}$.

3) $\left(\frac{5}{3}\right)^{x+2} = \left(\frac{27}{125}\right)^{2x+3}$

$\left(\frac{5}{3}\right)^{x+2} = \left(\frac{3}{5}\right)^{3(2x+3)}$

$\left(\frac{5}{3}\right)^{x+2} = \left(\frac{5}{3}\right)^{-3(2x+3)}$

$$x + 2 = -3(2x + 3)$$
$$x + 2 + 6x + 9 = 0$$
$$7x + 11 = 0$$

Answer: $x = -\frac{11}{7}$.

Check: $\left(\frac{5}{3}\right)^{-\frac{11}{7}+2} = \left(\frac{5}{3}\right)^{\frac{3}{7}} = \left(\frac{3}{5}\right)^{-\frac{3}{7}} = \left(\frac{3}{5}\right)^{3\left(-\frac{1}{7}\right)} = \left(\frac{27}{125}\right)^{-\frac{1}{7}} = \left(\frac{27}{125}\right)^{2\left(-\frac{11}{7}\right)+3}$.