Answer on Question #54752-Math-Statistics and Probability

Fred is answering a multiple choice question exam. Each question has n possible answers. If Fred knows the correct answer to a question, he always gets it correct; otherwise he takes a guess, picking any answer with equal probability.

Let K be the event that Fred knows the answer to a question and let R be the event that Fred answers the question correctly. Given $P(K) = p$ what is P(R)?

Solution

$$P(K|R) = \frac{P(KR)}{P(R)} = \frac{P(R|K)P(K)}{P(R|K)P(K) + P(R|K^C)P(K^C)}.$$

We already know that

$$P(R|K) = 1.$$

He takes a guess, picking any answer with equal probability, so

$$P(R|K^C) = \frac{1}{n}.$$

Thus,

$$P(K|R) = \frac{p}{p + \frac{1}{n}(1-p)} = \frac{np}{1 + (n-1)p}.$$