

### Answer on Question #54735, Math / Statistics and Probability

Senior management of a consulting services firm is concerned about a growing decline in the firm's weekly number of billable hours. The firm expects each professional employee to spend at least 40 hours per week on work. In an effort to understand this problem better, management would like to estimate the standard deviation of the number of hours their employees spend on work-related activities in a typical week. Rather than reviewing the records of all the firm's full-time employees, the management randomly selected a sample of size 51 from the available frame. The sample mean and sample standard deviations were 48.5 and 7.5 hours, respectively.

Construct a 90% confidence interval for the standard deviation of the number of hours this firm's employees spend on work-related activities in a typical week.

#### Solution:

In given problem, we have the following original data: Sample size ( $n$ ) = 51, Standard deviation ( $s$ ) = 7.5 hours, mean = 48.5 hours.

For a sample size of  $n = 51$ , we will have  $df = n - 1 = 50$  degrees of freedom. Then, we have to find the critical values:  $\chi_{\frac{1-\alpha}{2}}^2 = \chi_{0.95,50}^2 \approx 34.764$  and  $\chi_{\frac{\alpha}{2}}^2 = \chi_{0.05,50}^2 \approx 67.505$ . (We determine the critical values from the Chi-Square Distribution Table <http://sites.stat.psu.edu/~mga/401/tables/Chi-square-table.pdf>).

Then, the confidence interval is:

$$\frac{(n-1)s^2}{\chi_{\frac{\alpha}{2}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\frac{1-\alpha}{2}}^2}$$

Now, we substitute the values into the formula noted above:

$$\frac{(51-1) \cdot (7.5)^2}{67.505} < \sigma^2 < \frac{(51-1) \cdot (7.5)^2}{34.764}$$

Simplify the obtained inequality:

$$41.66358048 < \sigma^2 < 80.90265792$$

Finally, we can calculate the confidence interval for the population standard deviation:

$$41.66358048 < \sigma^2 < 80.90265792$$

Then, taking square roots gives us the following:

$$6.455 < \sigma < 8.995$$

Thus, we are 90% confident that the standard deviation of the number of hours this firm's employees spend on work-related activities in a typical week are between 6.455 and 8.995 hours.