Answer on Question #54705, Math Calculus

Draw diagrams to show two vectors a and b, and the vectors a + b and a − b.

When is the magnitude of a + b less than that of a − b?

When is the magnitude of a + b equal to that of a − b?

When is |a + b| = |a| + |b|?

Solution

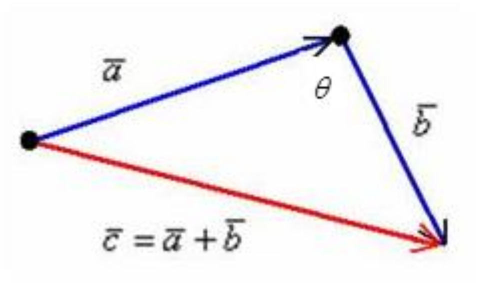

Fig.1 $\vec{a} + \vec{b}$

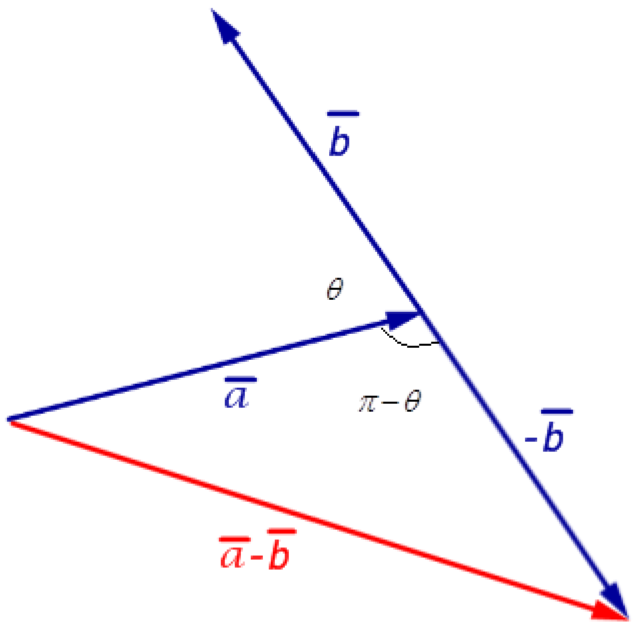

Fig.2 $\vec{a} - \vec{b}$

See Fig.1:

$$\left|\vec{a}+\vec{b}\right|=\sqrt{\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\vec{a}\cdot\vec{b}}=\sqrt{\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\theta)} \qquad (1)$$

where θ is the angle between vectors $\vec{a}$ and $+\vec{b}$.

See Fig.2:

$$\left|\vec{a}-\vec{b}\right|=\sqrt{\left|\vec{a}\right|^2+\left|-\vec{b}\right|^2-2\vec{a}\cdot\left(-\vec{b}\right)}=\sqrt{\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\pi-\theta)} \qquad (2)$$

$$\left|\vec{a}+\vec{b}\right|<\left|\vec{a}-\vec{b}\right|\Rightarrow\left|\vec{a}+\vec{b}\right|^2<\left|\vec{a}-\vec{b}\right|^2\Rightarrow\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\theta)<\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\pi-\theta)\Rightarrow$$
$$2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\theta)>2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\pi-\theta)\Rightarrow\cos(\theta)>\cos(\pi-\theta)\Rightarrow\cos(\theta)>-\cos(\theta)\Rightarrow2\cos(\theta)>0\Rightarrow$$
$$\cos(\theta)>0\Rightarrow\theta\in[0,\pi/2)$$

The magnitude of $\vec{a}+\vec{b}$ less than that of $\vec{a}-\vec{b}$ if $\theta\in[0,\pi/2)$.

$$\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|\Rightarrow\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}-\vec{b}\right|^2\Rightarrow\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\theta)=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\pi-\theta)\Rightarrow$$
$$2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\theta)=2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\pi-\theta)\Rightarrow\cos(\theta)=\cos(\pi-\theta)\Rightarrow\cos(\theta)=-\cos(\theta)\Rightarrow2\cos(\theta)=0\Rightarrow$$
$$\cos(\theta)=0\Rightarrow\theta=\pi/2$$

The magnitude of $\vec{a}+\vec{b}$ equal to that of $\vec{a}-\vec{b}$ if $\theta=\pi/2$

$$\left|\vec{a}+\vec{b}\right|^2=\left(\left|\vec{a}\right|+\left|\vec{b}\right|\right)^2\Rightarrow\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\theta)=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\Rightarrow2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cos(\theta)=$$
$$=2\left|\vec{a}\right|\cdot\left|\vec{b}\right|\Rightarrow\cos(\theta)=1\Rightarrow\theta=0$$

$\left|\vec{a}+\vec{b}\right|=\left|\vec{a}\right|+\left|\vec{b}\right|$ if vectors $\vec{a}$ and $\vec{b}$ are parallel.