

Answer on Question#54663, Math / Linear Algebra

For set of functions $\sin x, \cos x, x \cos x, x \sin x$ to be linearly independent, $a_0 \cos x + a_1 \sin x + a_2 x \cos x + a_3 x \sin x = 0$ only when $a_0 = 0, a_1 = 0, a_2 = 0, a_3 = 0$.

Let us evaluate $a_0 \cos x + a_1 \sin x + a_2 x \cos x + a_3 x \sin x = 0$ for $x = 0, x = \pi, x = \frac{\pi}{2}, x = \frac{\pi}{4}$.

Obtain:

$$\begin{aligned}x = 0: a_0 &= 0 \\x = \pi: -\pi a_2 &= 0 \\x = \frac{\pi}{2}: a_1 + \frac{\pi}{2} a_3 &= 0 \\x = \frac{\pi}{4}: \frac{a_1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} a_3 &= 0\end{aligned}$$

From first two equations, $a_0 = 0, a_2 = 0$, and substituting third equation into fourth, obtain $a_1 = 0$ and $a_3 = 0$.

Therefore, the set of functions $\sin x, \cos x, x \cos x, x \sin x$ are linearly independent.