

Answer on Question #54651 – Math – Calculus

Question:

$$\lim_{x \rightarrow \infty} (a^{1/x} - 1)x$$

Answer:

$$\lim_{x \rightarrow \infty} \left(\left(a^{\frac{1}{x}} - 1 \right) x \right) = \ln(a)$$

Steps

$$\lim_{x \rightarrow \infty} \left(\left(a^{\frac{1}{x}} - 1 \right) x \right)$$

$$\left(a^{\frac{1}{x}} - 1 \right) x = \frac{\left(a^{\frac{1}{x}} - 1 \right)}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow \infty} \left(\frac{a^{\frac{1}{x}} - 1}{\frac{1}{x}} \right)$$

Apply L'Hopital's Rule

$$= \lim_{x \rightarrow \infty} \left(\frac{-\frac{1}{a^x} \ln(a)}{-\frac{1}{x^2}} \right)$$

Refine

$$= \lim_{x \rightarrow \infty} \left(\frac{1}{a^x} \ln(a) \right)$$

Apply Infinity Properties

$$= a^0 \ln(a)$$

Simplify

$$= \ln(a)$$