Question #54617, Math / Differential Equations

please solve this problem

L^-1 { (1/s^2) e ^(-y*sqrt(s+a)) } .

Answer.

$$f(t) = L^{-1}\left\{\frac{1}{s^2}e^{-y\sqrt{s+a}}\right\}$$

$$L^{-1}\left\{e^{-y\sqrt{s+a}}\right\} = \frac{ye^{-at-\frac{y^2}{4t}}}{2\pi t^{\frac{3}{2}}}$$

As we know $L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(t)dt$

So $L^{-1}\left\{\frac{1}{s}e^{-y\sqrt{s+a}}\right\} = \int_0^t \frac{ye^{-at-\frac{y^2}{4t}}}{2\pi t^{\frac{3}{2}}}dt =$

$$= \frac{1}{2}\left[e^{-y\sqrt{a}}erfc\left(\frac{y-2\sqrt{a}}{2\sqrt{t}}\right) + e^{y\sqrt{a}}erfc\left(\frac{y+2\sqrt{a}}{2\sqrt{t}}\right)\right]$$

$$L^{-1}\left\{\frac{1}{s^2}e^{-y\sqrt{s+a}}\right\} = \int_0^t \frac{1}{2}\left[e^{-y\sqrt{a}}erfc\left(\frac{y-2\sqrt{a}}{2\sqrt{t}}\right) + e^{y\sqrt{a}}erfc\left(\frac{y+2\sqrt{a}}{2\sqrt{t}}\right)\right]dt =$$

$$= \frac{1}{4a}\left[(2at - y\sqrt{a})e^{-y\sqrt{a}}erfc\left(\frac{y-2\sqrt{a}}{2\sqrt{t}}\right) + (2at + y\sqrt{a})e^{y\sqrt{a}}erfc\left(\frac{y+2\sqrt{a}}{2\sqrt{t}}\right)\right]$$