Answer on Question #54582 - Math - Linear Algebra

Solve the linear equation:

$$\begin{cases} 2x + y - 3z = 5 \\ 3x - 2y - 2z = 5. \\ 5x - 3y - z = 16 \end{cases}$$

Solution

Gaussian elimination method.

1. Construct the augmented matrix for the system:

$$\left(\begin{array}{ccc|c} 2 & 1 & -3 & 5 \\ 3 & -2 & -2 & 5 \\ 5 & -3 & -1 & 16 \end{array}\right)$$

   Column 1 is for the coefficients of x.

   Column 2 is for the coefficients of y.

   Column 3 is for the coefficients of z.

   Row 1 contains first equation.

   Row 2 contains second equation.

   Row 3 contains third equation.

   The vertical line between the columns containing the coefficient and the column containing the constant term are there to show you that this is an augmented matrix.

2. We use elementary row operations to transform this matrix into a triangular one.

   We keep the first row and use it to produce all zeros elsewhere in the first column. We have:

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{3}{2} & \frac{5}{2} \\ 3 & -2 & -2 & 5 \\ 5 & -3 & -1 & 16 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{3}{2} & \frac{5}{2} \\ 0 & -\frac{7}{2} & \frac{5}{2} & -\frac{5}{2} \\ 0 & -\frac{11}{2} & \frac{13}{2} & \frac{7}{2} \end{array}\right).$$

   Next we keep the first and second row and try to have zeros in the second column. We get:

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{3}{2} & \frac{5}{2} \\ 0 & 1 & -\frac{5}{7} & \frac{5}{7} \\ 0 & -\frac{11}{2} & \frac{13}{2} & \frac{7}{2} \end{array}\right).$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -\frac{8}{7} & \frac{15}{7} \\ 0 & 1 & -\frac{5}{7} & \frac{5}{7} \\ 0 & 0 & \frac{18}{7} & \frac{52}{7} \end{array}\right).$$

Next we keep the first, second and third row and try to have zeros in the third column. We get:

$$\left(\begin{array}{ccc|c} 1 & 0 & -\frac{8}{7} & \frac{15}{7} \\ 0 & 1 & -\frac{5}{7} & \frac{5}{7} \\ 0 & 0 & 1 & \frac{26}{9} \end{array}\right).$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{49}{9} \\ 0 & 1 & 0 & \frac{25}{9} \\ 0 & 0 & 1 & \frac{26}{9} \end{array}\right).$$

This is a triangular matrix. Its associated system is:

$$\begin{cases} x & & & = & \frac{49}{9} \\ & y & & = & \frac{25}{9} \\ & & z & = & \frac{26}{9} \end{cases}.$$

Answer: $x = \frac{49}{9}, y = \frac{25}{9}, z = \frac{26}{9}$.