

Answer on Question#54581 - <Math> - < Linear algebra >

Solve the linear system

$$\begin{cases} 2x + y - 3z = 5 \\ 3x - 2y - 2z = 3 \\ 5x - 3y - 2z = 16 \end{cases}$$

Solution. Let's construct an augmented matrix with y variable for first column, x variable for second column and z variable for third column:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ -2 & 3 & -2 & 3 \\ -3 & 5 & -2 & 16 \end{array} \right]$$

Let's solve it by Gaussian elimination.

$$\begin{aligned} & \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ -2 & 3 & -2 & 3 \\ -3 & 5 & -2 & 16 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 7 & -8 & 13 \\ 0 & 11 & -11 & 31 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -8/7 & 13/7 \\ 0 & 11 & -11 & 31 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -8/7 & 13/7 \\ 0 & 0 & \frac{-77+88}{7} & \frac{217-143}{7} \end{array} \right] \sim \\ & \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -8/7 & 13/7 \\ 0 & 0 & 11/7 & 74/7 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -8/7 & 13/7 \\ 0 & 0 & 1 & 74/11 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & 0 & \frac{143+592}{77} \\ 0 & 0 & 1 & 74/11 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & 0 & 105/11 \\ 0 & 0 & 1 & 74/11 \end{array} \right] \sim \\ & \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & \frac{55-210}{11} \\ 0 & 1 & 0 & 105/11 \\ 0 & 0 & 1 & 74/11 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & \frac{-155}{11} \\ 0 & 1 & 0 & 105/11 \\ 0 & 0 & 1 & 74/11 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-155+222}{11} \\ 0 & 1 & 0 & 105/11 \\ 0 & 0 & 1 & 74/11 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 67/11 \\ 0 & 1 & 0 & 105/11 \\ 0 & 0 & 1 & 74/11 \end{array} \right] \end{aligned}$$

Thus, we obtain the solution:

$$\begin{cases} x = \frac{105}{11} \\ y = \frac{67}{11} \\ z = \frac{74}{11} \end{cases}$$

Answer: $\left(\frac{105}{11}, \frac{67}{11}, \frac{74}{11} \right)$