Answer on Question#54580 – Math – Linear Algebra

1. Find x by the use of determinant:

$$\begin{cases} 3x-4y+2z+8=0 \\ x+5y-3z+2=0 \\ 5x+3y-z+6=0 \end{cases}$$

Solution.

At first, we will rewrite our system of equations in matrix form:

$$\begin{pmatrix} 3 & -4 & 2 \\ 1 & 5 & -3 \\ 5 & 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} -8 \\ -2 \\ -6 \end{pmatrix}.$$

Now we will find the determinant of the coefficient matrix $\begin{pmatrix} 3 & -4 & 2 \\ 1 & 5 & -3 \\ 5 & 3 & -1 \end{pmatrix}$:

$$\Delta = det\begin{vmatrix} 3 & -4 & 2 \\ 1 & 5 & -3 \\ 5 & 3 & -1 \end{vmatrix} = -3\cdot 5+4\cdot 3\cdot 5+2\cdot 3-2\cdot 5\cdot 5+3\cdot 3\cdot 3-4=-15+60+6-$$

$$-50+27-4=24.$$

To find x we must know the determinant of the matrix which obtained from coefficient matrix by replacing column which correspond to x variable by column from right side of system:

$$\Delta_x = \det\begin{vmatrix} -8 & -4 & 2 \\ -2 & 5 & -3 \\ -6 & 3 & -1 \end{vmatrix} = 8\cdot 5-4\cdot 3\cdot 6-2\cdot 2\cdot 3+2\cdot 5\cdot 6-8\cdot 3\cdot 3+4\cdot 2=$$

$$=40-72-12+60-72+8=-48.$$

And now, from Cramer's rule we can find x:

$$x=\frac{\Delta_x}{\Delta}=-\frac{48}{24}=-2.$$

Answer:

hence, $x=-2$.