

## Answer on Question# 54574– Mathematics – Differential Equations

### Question:

If  $x=3t^{-1}, y=t^{-t}$ , then  $dy/dx$  is equal to...

### Answer:

**Definition of parametric differentiation:** if  $x = x(t)$  and  $y = y(t)$  then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{provided } \frac{dx}{dt} \neq 0. \quad (1)$$

1) If according to the statement of the problem we have

$$\begin{cases} x(t) = 3t^{-1}, \\ y(t) = t^{-t}. \end{cases} \quad (2)$$

then using (1) we obtain

$$\frac{dx}{dt} = 3 \cdot (-1)t^{-2} = -\frac{3}{t^2};$$

$$\ln(y) = -t \ln(t) \Rightarrow \frac{1}{y} \frac{dy}{dt} = -\ln(t) - \frac{t}{t} \Rightarrow \frac{dy}{dt} = -y(\ln(t) + 1) = -t^{-t}(\ln(t) + 1);$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-t^{-t}(\ln(t) + 1)}{-\frac{3}{t^2}} = \frac{1}{3} t^{2-t}(\ln(t) + 1);$$

$$\frac{dy}{dx} = \frac{1}{3} t^{2-t}(\ln(t) + 1). \quad (3)$$

2) If according to the statement of the problem we have

$$3) \begin{cases} x(t) = 3t^m - 1, \\ y(t) = t^n - t, \end{cases} \quad (4)$$

then we receive

$$\frac{dx}{dt} = 3mt^{m-1}; \quad \frac{dy}{dt} = nt^{n-1} - 1;$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{nt^{n-1} - 1}{3mt^{m-1}}. \quad (5)$$