

Answer on Question #54517 – Math – Differential Equations

pls solve inerse laplace problem :

$$L^{-1}\{a/(s+b)*\exp(-y*\sqrt{s+c})\}.$$

Solution

$$f(t) = L^{-1}\left\{\frac{a}{s+b}e^{-y\sqrt{s+c}}\right\}$$

$$L^{-1}\left\{\frac{a}{s}e^{-y\sqrt{s+c}}\right\} = \frac{a}{2}e^{-y\sqrt{c}}\operatorname{erfc}\left(\frac{y-2\sqrt{ct}}{2\sqrt{t}}\right) + \frac{a}{2}e^{y\sqrt{c}}\operatorname{erfc}\left(\frac{y+2\sqrt{ct}}{2\sqrt{t}}\right), \quad y > 0.$$

If $f(t) = L^{-1}\{F(s)\}$, then $f(t) = e^{at}L^{-1}\{F(s+a)\}$.

$$\text{Thus } f(t) = e^{bt}L^{-1}\left\{\frac{a}{s}e^{-y\sqrt{s+c-b}}\right\} =$$

$$= \frac{a}{2}e^{bt}\left[e^{-y\sqrt{c-b}}\operatorname{erfc}\left(\frac{y-2\sqrt{c-bt}}{2\sqrt{t}}\right) + e^{y\sqrt{c-b}}\operatorname{erfc}\left(\frac{y+2\sqrt{c-bt}}{2\sqrt{t}}\right)\right]$$