Answer on Question #54504 – Math – Linear Algebra

Solve the set of linear equations by Gaussian elimination method:

$$\begin{cases} a+3b+2c=3 \\ 2a-b-3c=-8 \\ 5a+2b+c=9 \end{cases}$$

Solve for c.

Solution

At first we will rewrite our system of equations in the form of matrix:

$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 2 & -1 & -3 & -8 \\ 5 & 2 & 1 & 9 \end{pmatrix}$, where fourth column is the right side of system of equations. Now, using Gaussian method we will transform our matrix into an upper triangular form.

1. The first row does not change. Multiplying the first row by (-2) and adding to the second row, the result will be placed in the second row:
$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 2 & -1 & -3 & -8 \\ 5 & 2 & 1 & 9 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & -7 & -7 & -14 \\ 5 & 2 & 1 & 9 \end{pmatrix}$;
2. Multiplying the first row by (-5) and adding to the third row, the result will be placed in the third row:
$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & -7 & -7 & -14 \\ 5 & 2 & 1 & 9 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & -7 & -7 & -14 \\ 0 & -13 & -9 & -6 \end{pmatrix}$;
3. Multiplying the second row by $\left(-\frac{1}{7}\right)$:
$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & -7 & -7 & -14 \\ 0 & -13 & -9 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -13 & -9 & -6 \end{pmatrix}$;
4. Multiplying the second row by 13 and adding to the third row, the result will be placed in the third row:
$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -13 & -9 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 20 \end{pmatrix}$;
5. Multiplying the third row by $\frac{1}{4}$:
$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 20 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 5 \end{pmatrix}$.

And now we can write our transformed system:

$$\begin{cases} a+3b+2c=3 \\ b+c=2 \\ c=5 \end{cases}$$

We see that $c=5$.

Answer: $c=5$.