

Answer on Question #54463– Math – Trigonometry

Question

A parcel of land has sides measuring 175ft, 234ft, 295ft and 415ft and the angle between the sides of the length 234ft and 295ft has measure 137.1° what is the measure of the opposite this angle?

Solution

According to the statement of the problem, the parcel of land has the quadrangular form (fig. 1).

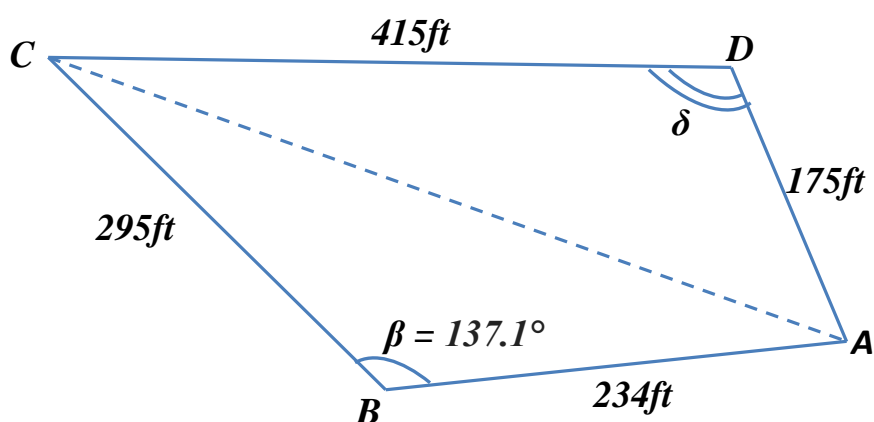


Fig. 1

We will find the angle δ , which is opposite to the angle $\beta = 137.1^\circ$, in a few steps.

At first we find the length of diagonal AC by using the law of cosines in triangle ABC:

$$(AC)^2 = (AB)^2 + (BC)^2 - 2AB \cdot BC \cdot \cos\beta. \quad (1)$$

Then using the law of cosines in triangle ADC, we find the cosine of angle δ :

$$(AC)^2 = (AD)^2 + (DC)^2 - 2AD \cdot DC \cdot \cos\delta; \Rightarrow$$
$$\cos\delta = \frac{(AD)^2 + (DC)^2 - (AC)^2}{2AD \cdot DC}. \quad (2)$$

Now, substituting (1) into (2) we obtain the general formula

$$\cos\delta = \frac{(AD)^2 + (DC)^2 - (AB)^2 - (BC)^2 + 2AB \cdot BC \cdot \cos\beta}{2AD \cdot DC}. \quad (3)$$

The substitution of all measures in (3) gives

$$\cos\delta = \frac{(175)^2 + (415)^2 - (234)^2 - (295)^2 + 2 \cdot 234 \cdot 295 \cdot \cos(137.1^\circ)}{2 \cdot 175 \cdot 415} = -0.27584. \quad (4)$$

And finally we use the inverse cosine, to find angle δ :

$$\delta = \arccos(-0.27584) = 106.06815^\circ \approx 106.1^\circ. \quad (5)$$