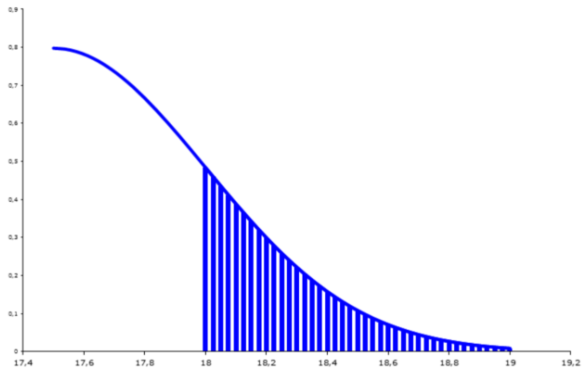
Answer on Question #54355 – Math – Statistics and Probability

A car manufacturer takes an average of 17.5 hours to construct a car. This includes time for stamping, welding, painting, assembly and inspections. Construction times vary with a standard deviation of 30 minutes and these times follow a normal distribution.

a. Find the construction time of a car, which is on the 10th percentile of this distribution.
b. What is the probability that a randomly selected car manufactured at this plant takes between 18 and 19 hours to construct?
c. Find the probability that the construction time for a randomly selected car manufactured at this plant is less than 17 hours and 50 minutes.

Solution



a.

Method 1

We need to find *a* such that

$$P(X < a) = 0.1,$$

where $X \sim N\left(17.5;\ \left(\frac{30}{60}\right)^2\right)$ is a random normally distributed variable.

Using Microsoft Excel 2013 type

=NORM.INV(0,1; 17,5; 0,5)

and the answer is 16.859.

Method 2

We need to find *a* such that

$$P(X < a) = 0.1,$$

where $X \sim N\left(17.5;\ \left(\frac{30}{60}\right)^2\right)$ is a random normally distributed variable.
Using statistical tables or the command

=NORM.S.INV(0,1)

in Microsoft Excel 2013 obtain that 10th percentile of the standard normal variable is -1.282, $P(Z < -1.282) = 0.1$.

It is known that

$$Z = \frac{X-E(X)}{sd(X)},$$

where $X \sim N\left(17.5;\ \left(\frac{30}{60}\right)^2\right)$, $Z \sim N(0;\ 1)$ are two random normally distributed variables.

Given times follow a normal distribution with the average of *E(X)*=17.5 hours and the standard deviation of $sd(X) = \frac{30}{60} = 0.5$ hour, equality

$$P(Z < -1.282) = 0.1$$

is equivalent to

$$P\left(\frac{X-E(X)}{sd(X)} < -1.282\right) = 0.1,$$

or

$$P(X < -1.282sd(X) + E(X)) = 0.1,$$

i.e.

$$P(X < -1.282 \cdot 0.5 + 17.5) = 0.1,$$
$$P(X < 16.859) = 0.1.$$

Thus, the construction time of a car, which is on the 10th percentile of this distribution, is 16. 859 hours.

b.

Method 1

We need to find probability

$$P(18 < X < 19),$$

where $X \sim N\left(17.5;\ \left(\frac{30}{60}\right)^2\right)$ is a random normally distributed variable.

Type

=NORM.DIST(19;17,5;0,5;TRUE)-NORM.DIST(18;17,5;0,5;TRUE)

in Microsoft Excel 2013 and the answer is 0.1573.

Method 2

It is known that

$$Z = \frac{X-E(X)}{sd(X)},$$

where $X \sim N\left(17.5;\ \left(\frac{30}{60}\right)^2\right)$, $Z \sim N(0;\ 1)$ are two random normally distributed variables.

Given times follow a normal distribution with the average of *E(X)*=17.5 hours and the standard deviation of $sd(X) = \frac{30}{60} = 0.5$ hour, the probability that a randomly selected car manufactured at this plant takes between 18 and 19 hours to construct is

$$P(18 < X < 19) = P\left(\frac{18-E(X)}{sd(X)} < \frac{X-E(X)}{sd(X)} < \frac{19-E(X)}{sd(X)}\right) = P\left(\frac{18-17.5}{0.5} < Z < \frac{19-17.5}{0.5}\right) =$$
$$= P(1 < Z < 3) = P(Z < 3) - P(Z < 1).$$

From z-table or using

=NORM.S.DIST(1;TRUE)

and

=NORM.S.DIST(3;TRUE)

in Microsoft Excel 2013 we know

$$P(Z < 1) = 0.84135;\ P(Z < 3) = 0.99865.$$

Thus,

$$P(18 < X < 19) = 0.9987 - 0.8413 = 0.1573.$$

c.

Method 1

We need to find probability $P\left(X < 17\,\frac{50}{60}\right)$, where $X \sim N\left(17.5;\ \left(\frac{30}{60}\right)^2\right)$ is a random normally distributed variable.

Type

=NORM.DIST(17+50/60;17,5;0,5;TRUE)

in Microsoft Excel 2013 and the answer is 0.7475.

Method 2

It is known that

$$Z = \frac{X - E(X)}{sd(X)},$$

where $X \sim N\left(17.5;\ \left(\frac{30}{60}\right)^2\right)$, $Z \sim N(0;\ 1)$ are two random normally distributed variables.

Given times follow a normal distribution with the average of *E(X)*=17.5 hours and the standard deviation of $sd(X) = \frac{30}{60} = 0.5$ hour, the probability that the construction time for a randomly selected car manufactured at this plant is less than 17 hours and 50 minutes will be

$$P\left(X < 17\,\frac{50}{60}\right) = P\left(\frac{X - E(X)}{sd(X)} < \frac{17\frac{50}{60} - E(X)}{sd(X)}\right) = P\left(Z < \frac{17\frac{50}{60} - 17.5}{0.5}\right) =$$

$$= P\left(Z < \frac{17\frac{50}{60} - 17\frac{30}{60}}{0.5}\right) = P\left(Z < \frac{20}{60 \cdot 0.5}\right) = P\left(Z < \frac{20}{30}\right) = P(Z < 0.667) = 0.7475.$$

From z-table we know

$$P(Z < 0.66) = 0.7454;\ P(Z < 0.67) = 0.7486.$$

Type

=NORM.S.DIST(2/3;TRUE)

in Microsoft Excel 2013 and the answer is 0.7475.

Answer:

a. 16.859;

b. 0.1573;

c. $\mathbf{0.7475}$.