Answer on Question #54325-Math-Statistics and Probability

Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the element of the sample space S for the three tosses of the coin and to each sample point assign a value of W.

Solution

For 3 tosses, possibilities are:

$$0\ heads \rightarrow 3\ tails \rightarrow W = (heads - tails) = 0 - 3 = -3$$

$$1\ head \rightarrow 2\ tails \rightarrow W = (heads - tails) = 1 - 2 = -1$$

$$2\ heads \rightarrow 1\ tail \rightarrow W = (heads - tails) = 2 - 1 = 1$$

$$3\ heads \rightarrow 0\ tails \rightarrow W = (heads - tails) = 3 - 0 = 3$$

So, the elements of the sample space are:

$$S = \{-3, -1, 1, 3\}.$$

To find the probability for each case:

$$P(0\ heads\ \&\ 3\ tails) = \frac{3!}{0!\,(3-0)!}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(1\ head\ \&\ 2\ tails) = \frac{3!}{1!\,(3-1)!}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P(2\ heads\ \&\ 1\ tail) = \frac{3!}{2!\,(3-2)!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8}$$

$$P(3\ heads\ \&\ 0\ tails) = \frac{3!}{3!\,(3-3)!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = \frac{1}{8}$$

i.e.:

$$For\ W = -3 \rightarrow P = 1/8$$

$$For\ W = -1 \rightarrow P = 3/8$$

$$For\ W = 1 \rightarrow P = 3/8$$

$$For\ W = 3 \rightarrow P = 1/8$$