

Answer on Question #54278 – Math – Calculus

Question

Each side of a square is increasing at a rate of 7 cm/s. At what rate is the area of the square increasing when the area of the square is 64 cm²?

Solution

Let A = Area; a = side of the square.

It is known that $A = a^2$. If the area of the square is $A = 64 \text{ cm}^2$, then the side of the square is $a = 8 \text{ cm}$.

Since A is also a function of time:

$$A(t) = a^2(t),$$

so the rate of change in A(t) with respect to time t is the following:

$$\frac{dA}{dt} = \frac{dA}{da} \cdot \frac{da}{dt} \text{ due to the chain rule.}$$

Differentiating,

$$\frac{dA}{dt} = \frac{dA}{da} \cdot 7 = 2a \text{ cm} \cdot 7 \text{ cm/s} = 14a \text{ cm}^2/\text{s}.$$

Therefore, the rate at what the area is increasing, when $a = 8$, is

$$\left. \frac{dA}{dt} \right|_{a=8 \text{ cm}} = (14 \text{ cm/s})(8 \text{ cm}) = \mathbf{112 \text{ cm}^2/\text{s}}.$$

$$\mathbf{\text{Answer: } 14a \frac{\text{cm}^2}{\text{s}} = 112 \frac{\text{cm}^2}{\text{s}}.}$$