Answer on Question #54160 – Math – Calculus

Solve the initial value problems:

a) y' = (x y sin(x))/(y+1), y(0) = 1

b) t(du/dt) = t^2 + 3u, t>0, u(2)=4

Solution

a) $y' = \frac{xysinx}{y+1},\ y(0) = 1.$

$\frac{y+1}{y}dy = xsinx\ dx \ \ \rightarrow \ \ \int \frac{y+1}{y}dy = \int xsinx\ dx \ \ \rightarrow$

$\rightarrow \int \left(1 + \frac{1}{y}\right) dy = -\int x\ dcos(x) \rightarrow$

$\rightarrow y + lny = -(xcos(x) - \int cos(x)dx) + C \ \rightarrow$

$\rightarrow \ \ y + lny = -xcosx + sinx + C;$ (1)

$y(0) = 1$ (2)

Equalities (1) and (2) give the following expression:

$y(0) + lny(0) = -0 \cdot cos(0) + sin(0) + C \rightarrow$

$1 = C$ (3)

Substitute (3) for *C* in (1) and obtain the answer.

Solution to the initial value problem is

$$\boldsymbol{y + lny = -xcosx + sinx + 1}.$$

b) $t\frac{du}{dt} = t^2 + 3u,\ t > 0, u(2) = 4.$

Rewrite $t\frac{du}{dt} = t^2 + 3u$ as

$\frac{du}{dt} - \frac{3}{t}u = t$ (4)

$\frac{du}{dt} + P(t)u = f(t) \quad where \;\; P(t) = -\frac{3}{t},\; f(t) = t$

Integrating factor is the following:

$e^{\int P(t)dt} = e^{-3lnt} = t^{-3}$ (5)

Multiply (4) by (5) :

$$\frac{1}{t^3}\frac{du}{dt} - \frac{3}{t^4}u = \frac{t}{t^3}$$

$$\frac{d}{dt}\left(\frac{u}{t^3}\right) = \frac{1}{t^2}$$

Integrate both sides with respect to t :

$\frac{u}{t^3} = \int \frac{dt}{t^2} \;\rightarrow\; \frac{u}{t^3} = -\frac{1}{t} + C \;\rightarrow$

Multiply both sides by t^3:

$u = Ct^3 - t^2;$ (6)

It follows from the initial condition $u(2) = 4$ and (6) that

$u(2) = 4 \rightarrow \;\; 4 = 8C - 4 \;\rightarrow$

$C = 1$ (7)

Substitute (7) for C in (6) and obtain the answer.

Solution to the initial value problem is

$\boldsymbol{u = t^3 - t^2}$.