Answer on Question #54159 – Math – Calculus

Establish, by using limits, integration by parts and I'Hopitals Rule, whether the following integral converges: $\int_0^1 \ln x dx$.

Solution

Let's find antiderivative of function $f(x) = \ln x$, integral $\int \ln x dx$ using integration by parts:

$$F(x) + CONST = \int \ln x dx = \left\{ \begin{matrix} u = \ln x \\ dv = dx \end{matrix} \middle| \begin{matrix} du = \frac{dx}{x} \\ v = x \end{matrix} \right\} = x \ln x - \int \frac{x dx}{x} = x \ln x - x + CONST ,$$

where *CONST* is an arbitrary real constant of integration.

Thus,

$$\int_0^1 \ln x dx = \lim_{x\to 1}(x \ln x - x + CONST) - \lim_{x\to 0}(x \ln x - x + CONST) = \ln 1 - 1 + CONST - \lim_{x\to 0} x \ln x + 0 - CONST =$$

$$= -1 - \lim_{x\to 0} \frac{\ln x}{x^{-1}} = \{ \text{ due to L'Hopital's rule, there is an indeterminate form of type } \frac{\infty}{\infty} \} =$$

$$= -1 - \lim_{x\to 0} \frac{(\ln x)'}{(x^{-1})'} = -1 - \lim_{x\to 0} \frac{x^{-1}}{-x^{-2}} = -1 - \lim_{x\to 0} \frac{-x^2}{x} = -1 - \lim_{x\to 0}(-x) = -1 .$$

Hence, integral $\int_0^1 \ln x dx$ converges to -1.

Answer: $\int_0^1 \ln x dx = -1$.