Answer on Question #54157 – Math – Calculus

Without using L'Hopital's Rule, evaluate the $\lim_{t \to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$.

Solution

Method 1 (without using L'Hopital's rule)

Substituting for $t = 0$ numerator and denominator will give 0, hence we get indeterminate form of type 0/0. To evaluate the limit, we shall multiply the numerator and the denominator of our expression by $\sqrt{1+t} + \sqrt{1-t}$:

$$\lim_{t \to 0} \frac{(\sqrt{1+t}-\sqrt{1-t})(\sqrt{1+t}+\sqrt{1-t})}{t(\sqrt{1+t}+\sqrt{1-t})}.$$

In the numerator we shall use the next formula: $(x-y)(x+y) = x^2 - y^2$.

Now we can rewrite our expression as follows:

$$\lim_{t \to 0} \frac{(\sqrt{1+t}-\sqrt{1-t})(\sqrt{1+t}+\sqrt{1-t})}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t \to 0} \frac{1+t-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t \to 0} \frac{2t}{t(\sqrt{1+t}+\sqrt{1-t})} =$$

$$= \lim_{t \to 0} \frac{2}{\sqrt{1+t}+\sqrt{1-t}} = \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{1+1} = 1.$$

Answer: $\lim_{t \to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = 1.$

Method 2 (using L'Hopital's rule)

By the way, L'Hopital's rule gives the same answer, because

$$\lim_{t \to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = \left|\frac{0}{0}\right| = \lim_{t \to 0} \frac{(\sqrt{1+t}-\sqrt{1-t})'}{(t)'} = \lim_{t \to 0} \frac{\frac{1}{2\sqrt{1+t}} - \frac{1}{2\sqrt{1-t}} \cdot (-1)}{1}$$

$$= \lim_{t \to 0} \left(\frac{1}{2\sqrt{1+t}} + \frac{1}{2\sqrt{1-t}}\right) = \frac{1}{2\sqrt{1+0}} + \frac{1}{2\sqrt{1-0}} = \frac{1}{2} + \frac{1}{2} = 1$$

Answer: $\lim_{t \to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = 1.$