Answer on Question #54121 – Math – Combinatorics | Number Theory

The number of ways 4 letters can be put into three letter boxes.

Solution

Assume that the letters 1, 2, 3, 4 and letter boxes A, B, C are marked in some way or have some feature about them that makes each one distinguishable from the others, but with no restriction on the number of letters that can go into each letter box.

Letter 1 can be put in A, B, or C letter boxes, i.e. in 3 ways.

Letter 2 can be put in A, B, or C letter boxes, i.e. in 3 ways.

Letter 3 can be put in A, B, or C letter boxes, i.e. in 3 ways.

Letter 4 can be put in A, B, or C letter boxes, i.e. in 3 ways.

Here we deal with ordered permutations of the boxes, but with unrestricted repetitions. We have 3 options to arrange each letter, so by the fundamental principle of counting, the total number of ways of putting $k = 4$ letters into $n = 3$ letter boxes is

$$\underbrace{n \cdot n \cdot \ldots \cdot n}_{k} = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4 = n^k = 81.$$

Answer: 81.

In general when we speak of ***k* distinguishable letters**, we will assume that they are numbered with the consecutive integers 1 through *k*. When we speak of ***n* distinguishable boxes**, we will assume that they are numbered with the consecutive integers 1 through *n*. The term **"indistinguishable"** refers to the fact that the balls, or boxes, are so identical that there is no way to tell them apart. When placing indistinguishable letters into distinguishable boxes all that we are able to tell is the number of letters that end up in each box. The phrase **"with exclusion"** means that no box can contain more than one ball. The phrase **"without exclusion"** means that a box may contain more than one letter.

Letters	**Boxes**	**Exclusion**	**No box empty**	**Number of ways of putting *k* letters into *n* boxes**
Dist	**Dist**	**with**		***n*(*n-1*)(*n-2*)…(*n-k+1*)**
Dist	**Dist**	**with**	**yes**	**n! if *k=n* 0 if k$\neq n$**
Dist	**Dist**	**without**		n^k
Dist	**Dist**	**without**	**yes**	$\sum_{i=0}^{n-1}(-1)^i \binom{n}{i}(n-i)^k$
Indist	**Dist**	**with**		$\binom{n}{k}$
Indist	**Dist**	**with**	**yes**	**1 if *k=n* 0 if $k \neq n$**
Indist	**Dist**	**without**		$\binom{n+k-1}{k}$
Indist	**Dist**	**without**	**yes**	$\binom{k-1}{n-1}$

where Dist=distinguishable, Indist=indistinguishable, with= with exclusion, without=without exclusion,

$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n, \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}$.