Question

The nth term of a sequence is given. Determine whether the sequence converges or diverges. If it converges find its limit

(i) n1/n

(ii) 2n/(2n)!

(iii) en/n

(iv) 1+1/2!+1/3!+………..+1/n!

Solution

(i) $\frac{n!}{n} = (n-1)!$ **diverges.**

(ii) $\frac{2n}{(2n)!} = \frac{1}{(2n-2)!}$ **converges,** $\lim\limits_{n\to\infty} \frac{1}{(2n-2)!} = 0$.

(iii) $\frac{e^n}{n}$ **diverges.**

(iv) $1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$ **converges,**

$$\lim\limits_{n\to\infty}\left(1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}\right) = \lim\limits_{n\to\infty}\sum_{k=1}^{n}\frac{1}{k!} = \sum_{k=1}^{\infty}\frac{1}{k!} = e - 1.$$