Answer on Question #54046 – Math – Statistics and Probability

In an effort to estimate the annual income for new graduates, data were collected from 100 new graduates over a 1-year period. Assume a population standard deviation of $2,000. If the sample mean is $15,000, what is the 95% confidence interval for the population mean?

Select one:

a. $14,608 to $15,392

b. $14,668 to $15,332

c. $14,590 to $15,410

d. $14,534 to $15,466

e. $14,603 to $15,396

Solution

In the given problem we have the following data: population standard deviation, $SD = \$2\ 000$, sample mean, $\bar{x} = \$15\ 000$, sample size, N = 100 graduates. We need to determine the 95% confidence interval for the population mean.

The formula for calculating the standard error of the mean is

$$\mathrm{s} = \frac{\mathrm{SD}}{\sqrt{\mathrm{N}}}$$

Where s= the standard error

SD = the standard deviation of the sample

N = the sample size

Thus, we can substitute the given values into the formula:

$$\mathrm{s} = \frac{\mathrm{SD}}{\sqrt{\mathrm{N}}} = \frac{\$2000}{\sqrt{100}} = \frac{\$2000}{10} = \$200$$

Since $200 is the standard deviation of our given theoretical sampling distribution, approximately 95% of that distribution falls between the means of $14 600 and $15 400, it is explaining by the fact that, $14 600 is two standard deviations $(2 \cdot \$200 = \$400)$ below the mean of $15 000 and $15 400 is two standard deviations above $15 000.

A 95% confidence interval is a range of values that we can be 95% certain contains the true mean of the population. We use the following formula for calculations:

95% confidence interval = $\left(\bar{\mathrm{x}} - \left(\mathrm{z}_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{\mathrm{N}}}\right); \bar{\mathrm{x}} + \left(\mathrm{z}_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{\mathrm{N}}}\right)\right)$

95% confidence interval = $(\bar{x} - 1.96 \cdot s; \bar{x} + 1.96 \cdot s)$

Thus, we can substitute the values into the formula noted above.

$$(\$15\ 000 - 1.96 \cdot \$200; \$15\ 000 + 1.96 \cdot \$200)$$

We simplify the obtained expression:

$$(\$15\ 000 - \$392; \$15\ 000 + \$392)$$

Finally, we can be 95% confident that the true mean income for graduates is somewhere between \$14 608 and \$15 392. Thus, we have a confidence level of 95% that the population mean falls within the confidence interval of \$14 608 and \$15 392.

The answer is a. \$14,608 to \$15,392.

Question 9

If the sample mean is \$15,000, what is the 98% confidence interval for the population mean?

Select one:

a. \$14,590 to \$15,410

b. \$14,608 to \$15,392

c. \$14,603 to \$15,396

d. \$14,668 to \$15,332

e. \$14,534 to \$15,466

Solution

In the given problem we have the following data: population standard deviation, $\mathrm{SD} = \$2\ 000$, sample mean, $\bar{x} = \$15\ 000$, sample size, N = 100 graduates. We need to determine the 98% confidence interval for the population mean. The standard error of the mean is equal to $\$200$, we have already determined it.

It should be noted that in this case we need to estimate the probability less than 0.05 of being wrong about our confidence interval. We need to construct a 98% confidence interval.

The level of confidence is 98% or Probability of 0.98, then $\alpha = 1 - \frac{98}{100} = 1 - 0.98 = 0.02$.

Then, we find the critical probability p:

$$p = 1 - \frac{\alpha}{2} = 1 - \frac{0.02}{2} = 1 - 0.01 = 0.99$$

The critical value $z_{\frac{\alpha}{2}} = 2.33$

Now, we apply the formula for calculation the 98 % confidence interval for the population mean:

98% confidence interval =$\left(\bar{\mathrm{x}} - \left(2.33\frac{\sigma}{\sqrt{N}}\right); \bar{\mathrm{x}} + \left(2.33\frac{\sigma}{\sqrt{N}}\right)\right)$

Then, we can substitute the values into the formula noted above.

$$(\$15\,000 - 2.33 \cdot \$200;\ \$15\,000 + 2.33 \cdot \$200\,)$$

We simplify the obtained expression:

$$(\$15\,000 - \$466) \text{ to } (\$15\,000 + \$466)$$

Finally, we can be 98% confident that the true mean income for graduates is somewhere between \$14 534 and \$15 466. Thus, we have a confidence level of 98% that the population mean falls within the confidence interval of \$14 534 and \$15 466.

The answer is e. \$14,534 to \$15,466.