Answer on Question #54000 – Math – Statistics and Probability

Question

At a supermarket 80% of customers pay by credit card. Find the probability that a randomly selected sample of ten customers,

i) Exactly two pay by credit card

ii) Less than three pay by credit card.

Solution

Let ξ be a random variable of customers who pay by credit card. Obviously ξ has a binomial distribution with the following parameters: n = 10 independent attempts; p = 0.8 is a probability of success in one attempt.

i)
$$P\{\xi = 2\} = C_n^2 \cdot p^2 (1-p)^8 = \frac{10!}{2! \cdot 8!} \cdot 0.8^2 \cdot (0.2)^8 \approx 0.00007.$$

ii) $P\{\xi < 3\} = P\{\xi = 0\} + P\{\xi = 1\} + P\{\xi = 2\} = \sum_{k=0}^2 C_{10}^k p^k (1-p)^{10-k} = 0.00007.$

 $= \sum_{k=0}^{2} C_{10}^{k} (0.8)^{k} (0.2)^{10-k} = \frac{10!}{0! \cdot 10!} \cdot 0.8^{0} \cdot (0.2)^{10} + \frac{10!}{1! \cdot 9!} \cdot 0.8^{1} \cdot (0.2)^{9} + \frac{10!}{2! \cdot 8!} \cdot 0.8^{2} \cdot (0.2)^{8} \approx 0.0000779.$

Answer: i) 0.00007; ii) 0.0000779.