Answer on Question #53982 – Math - Calculus

Use the reduction formula to evaluate the integral

$$\int_1^e (\ln x)^3\, dx.$$

Solution:

We have

$$\int_1^e (\ln x)^n\, dx = \left| \begin{matrix} u = (\ln x)^n \Rightarrow du = n(\ln x)^{n-1}\dfrac{dx}{x} \\ dv = dx \Rightarrow v = x \end{matrix} \right| = x(\ln x)^n|_1^e - \int_1^e xn(\ln x)^{n-1}\frac{dx}{x} =$$

$$= x(\ln x)^n|_1^e - n\int_1^e (\ln x)^{n-1} dx.$$

Thus in our case $n = 3$ and we get

$$\int_1^e (\ln x)^3\, dx = x(\ln x)^3|_1^e - 3\int_1^e (\ln x)^2 dx = x(\ln x)^3|_1^e - 3\left(x(\ln x)^2|_1^e - 2\int_1^e \ln x\, dx\right) =$$

$$= x(\ln x)^3|_1^e - 3\left(x(\ln x)^2|_1^e - 2\left(x\ln x|_1^e - \int_1^e dx\right)\right) =$$

$$= e - 3\big(e - 2(e - x|_1^e)\big) = e - 3e + 6\big(e - (e - 1)\big) = 6 - 2e.$$

Answer: $\int_1^e (\ln x)^3\, dx = 6 - 2e.$