Answer on Question #53953, Math Real Analysis

prove that f(x) =x/(x squire+1),x is belongs to set of real number, is bijection.

Solution:

$$f(x) = \frac{x}{x^2 + 1}$$

If $x \in \mathbb{R} \rightarrow f(x) \in \mathbb{R}$.

$$\forall x_1 \in X, \forall x_2 \in X \Rightarrow f(x_1) = \frac{x_1}{x_1^2 + 1}; f(x_2) = \frac{x_2}{x_2^2 + 1}$$

$$\frac{x_1}{x_1^2 + 1} = \frac{x_2}{x_2^2 + 1} \Rightarrow \left(x_2^2 + 1\right)x_1 = \left(x_1^2 + 1\right)x_2 \Rightarrow$$

$$x_2^2 x_1 - \left(x_1^2 + 1\right)x_2 + x_1 = 0$$

$$D = \left(x_1^2 + 1\right)^2 - 4x_1^2 = \left(x_1^2 - 1\right)^2$$

$$x_2 = \frac{\left(x_1^2 + 1\right) \pm \left|x_1^2 - 1\right|}{2x_1}$$

If $x_1^2 - 1 > 0 \Rightarrow$

$$x_{2_{1,2}} = \frac{\left(x_1^2 + 1\right) \pm \left(x_1^2 - 1\right)}{2x_1}$$

$$x_{2_1} = x_1$$

$$x_{2_2} = 1/x_1$$

if $x_1^2 - 1 < 0 \Rightarrow$

$$x_{2_{1,2}} = \frac{\left(x_1^2 + 1\right) \pm \left(-x_1^2 + 1\right)}{2x_1}$$

$$x_{2_1} = 1/x_1$$

$$x_{2_2} = x_1$$