Answer on Question# 53949– Mathematics – Calculus

Question:

Graph each pair of parametric equations: x = 2t – 1, y = t2 + 5, -4 ≤ t ≤ 4

Answer:

Definition:

A curve in the *xy*-plane is said to be parameterized if the set of coordinates on the curve, *(x, y)*, are represented as functions of a variable *t*. Namely,

$$x = f(t), \;\; y = g(t), \;\; t \in D, \tag{1}$$

where *D* is a set of real numbers. The variable *t* is called a *parameter* and the relations between *x, y* and *t* are called *parametric equations*. The set *D* is called the domain of *f* and *g* and it is the set of values *t* takes.

According to the problem statement we have the following pair of parametric equations:

$$\begin{cases} x = f(t) = 2t - 1, \\ y = g(t) = t^2 + 5, \\ \quad -4 \le t \le 4. \end{cases} \tag{2}$$

There are several techniques we use to sketch a curve generated by a pair of parametric equations (2):

1) the evaluation of *f(t)* and g(t) for several values of *t* and plotting the points *(f(t), g(t))* in the *xy*-plane;
2) the elimination of the parameter *t* to find the explicit equation of *y* as a function of *x.*

1) To plot the graph of the required curve we use a table of values with values of *t* from *–4* to *4*:

t	*y=f(t)=2t-1*	*y=g(t)=t²+5*	*(x, y)*
-4	-9	21	(-9,21)
-3	-7	14	(-7,14)
-2	-5	9	(-5,9)
-1	-3	6	(-3,6)
0	-1	5	(-1,5)
1	1	6	(1,6)
2	3	9	(3,9)
3	5	14	(5,14)
4	7	21	(7,21)

Let's plot the points that are labeled as *(x, y)*-coordinates and connect them on the graph by the smooth curve (fig.1). Note that the orientation of a parameterized curve is the direction determined by increasing values of the parameter. In our case, the direction of *t* increasing is from left to right.

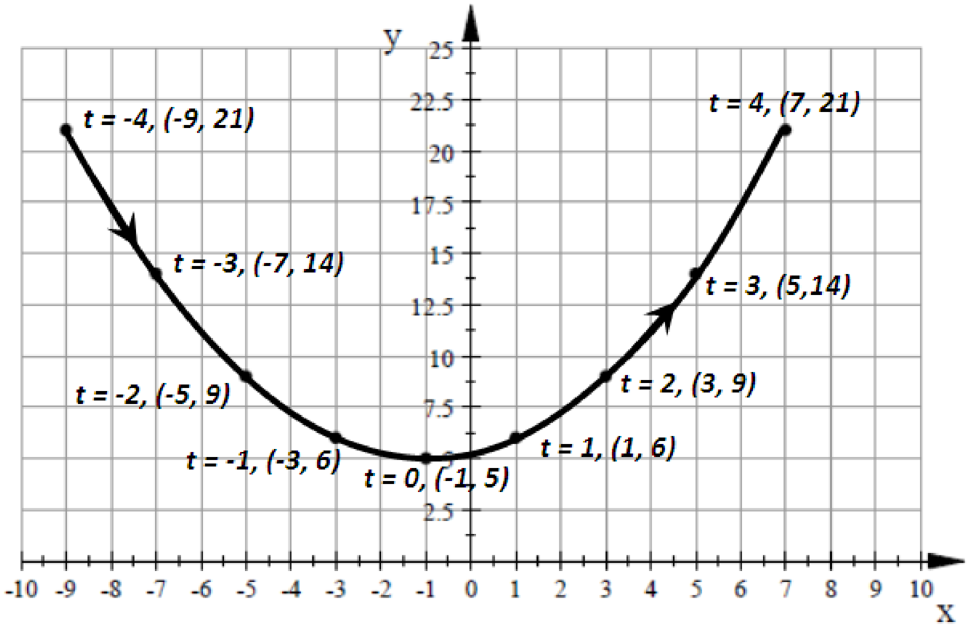

Fig.1

2) To plot the graph of the required curve we eliminate the parameter *t* from the equations (2).

Since $x = 2t–1$, then the solution for *t* in terms of *x* is

$$t = \frac{x+1}{2}. \quad (3)$$

Substituting (3) into the the equation for *y* to eliminate *t*, we get

$$y(x) = \frac{(x+1)^2}{4} + 5. \quad (4)$$

It is easy to see that the required curve is a parabola. The vertex of the parabola (4) is at the point *A (-1, 5)*, and the straight line $x = -1$ is the axis of its symmetry (fig.2).

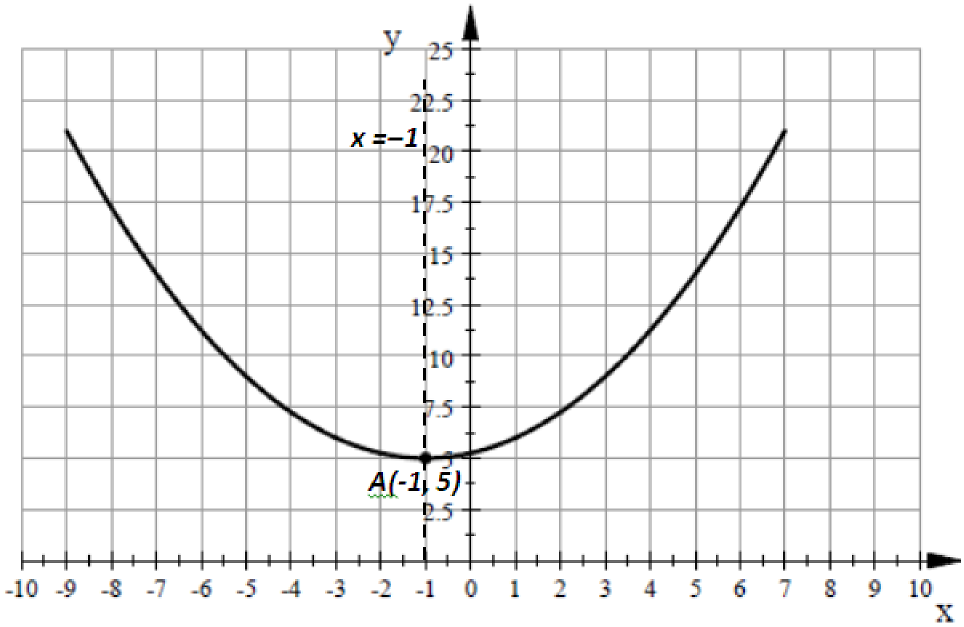

Fig.2

Thus, each of the presented techniques yields to the same result.