Answer on Question #53944– Math – Complex Analysis

Question

Find the fifth roots of 32(cos 280° + i sin 280°).

Solution

Definition:

Assume that the complex number *z* is presented in trigonometric form

$$z = \rho(cos\varphi + isin\varphi), \tag{1}$$

then the n-th roots of *z* are defined by the formula

$$\sqrt[n]{z} = \sqrt[n]{\rho}\left(cos\frac{\varphi+2\pi k}{n} + isin\frac{\varphi+2\pi k}{n}\right), \tag{2}$$

where *k = 0,1,2,…,n-1*.

In the given problem the complex number *z* has the form

$$z = 32(cos280^{\circ} + isin280^{\circ}). \tag{3}$$

For convenience, let's rewrite the polar angle φ in radians:

$$280^{\circ} = 280^{\circ} \cdot \frac{\pi}{180^{\circ}} = \frac{14}{9}\pi. \tag{4}$$

Hence, for (3) we obtain

$$z = 32\left(cos\left(\frac{14}{9}\pi\right) + isin\left(\frac{14}{9}\pi\right)\right). \tag{5}$$

Now using (2) we find the fifth roots of (5):

$$\sqrt[5]{z} = \sqrt[5]{32}\left(cos\frac{\frac{14}{9}\pi+2\pi k}{5} + isin\frac{\frac{14}{9}\pi+2\pi k}{5}\right), k = 0,1,2,3,4; \tag{6}$$

$$\text{for } k = 0: \quad z_1 = 2\left(cos\frac{\frac{14}{9}\pi + 0}{5} + isin\frac{\frac{14}{9}\pi + 0}{5}\right) = 2\left(cos\frac{14}{45}\pi + isin\frac{14}{45}\pi\right),$$

$$\text{for } k = 1: \quad z_2 = 2\left(cos\frac{\frac{14}{9}\pi + 2\pi}{5} + isin\frac{\frac{14}{9}\pi + 2\pi}{5}\right) = 2\left(cos\frac{32\pi}{45} + isin\frac{32\pi}{45}\right),$$

$$\text{for } k = 2: \quad z_3 = 2\left(cos\frac{\frac{14}{9}\pi + 4\pi}{5} + isin\frac{\frac{14}{9}\pi + 4\pi}{5}\right) = 2\left(cos\frac{10\pi}{9} + isin\frac{10\pi}{9}\right),$$

for $k = 3$: $z = 2\left(cos\frac{\frac{14}{9}\pi + 6\pi}{5} + isin\frac{\frac{14}{9}\pi + 6\pi}{5} \right) = 2\left(cos\frac{68\pi}{45} + isin\frac{68\pi}{45} \right),$

for $k = 4$: $z_5 = 2\left(cos\frac{\frac{14}{9}\pi + 8\pi}{5} + isin\frac{\frac{14}{9}\pi + 8\pi}{5} \right) = 2\left(cos\frac{86\pi}{45} + isin\frac{86\pi}{45} \right).$

Therefore, the fifth roots of $z = 32\left(cos\left(\frac{14}{9}\pi\right) + isin\left(\frac{14}{9}\pi\right) \right)$ are

$$\left\{\begin{array}{c} z_1 = 2\left(cos\frac{14}{45}\pi + isin\frac{14}{45}\pi \right),\ z = 2\left(cos\frac{32\pi}{45} + isin\frac{32\pi}{45} \right), \\ z_3 = 2\left(cos\frac{10\pi}{9} + isin\frac{10\pi}{9} \right),\ z_4 = 2\left(cos\frac{68\pi}{45} + isin\frac{68\pi}{45} \right), \\ z_5 = 2\left(cos\frac{86\pi}{45} + isin\frac{86\pi}{45} \right). \end{array}\right\} \quad (7)$$