Answer on Question #53939 – Math – Calculus

Question. Determine if the graph is symmetric about the x-axis, the y-axis, or the origin.

$r = 3\cos 5\theta$.

Solution. $\cos 5\theta = \cos 3\theta \cos 2\theta - \sin 3\theta \sin 2\theta = (2(\cos\theta)^2 - 1)(4(\cos\theta)^3 - 3\cos\theta) -$

$-2\sin\theta\cos\theta\,(3\sin\theta - 4(\sin\theta)^3) = 8(\cos\theta)^5 - 10(\cos\theta)^3 + 3\cos\theta -$

$$-(6(\sin\theta)^2\cos\theta - 8(\sin\theta)^4\cos\theta) =$$

$= 8(\cos\theta)^5 + 8(\sin\theta)^4\cos\theta - 10(\cos\theta)^3 - 6(\sin\theta)^2\cos\theta + 3\cos\theta$. Using the fact that

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ r = \sqrt{x^2+y^2} \end{cases} \Rightarrow \begin{cases} r = \sqrt{x^2+y^2} \\ \cos\theta = \dfrac{x}{\sqrt{x^2+y^2}} \\ \sin\theta = \dfrac{y}{\sqrt{x^2+y^2}} \end{cases}$$ we can rewrite the original equation in the next form:

$$\sqrt{x^2+y^2} = 3\left[\frac{8x^5}{(x^2+y^2)^{5/2}} + \frac{8y^4}{(x^2+y^2)^2}\cdot\frac{x}{\sqrt{x^2+y^2}} - \frac{10x^3}{(x^2+y^2)^{3/2}} - \frac{6y^2}{x^2+y^2}\cdot\frac{x}{\sqrt{x^2+y^2}} + \frac{3x}{\sqrt{x^2+y^2}}\right] (*).$$

It follows from the last equation that if the point $(x_0; y_0)$ belongs to the graph, then the point $(x_0; -y_0)$ also belongs to the graph. So the graph is symmetric about the x-axis.
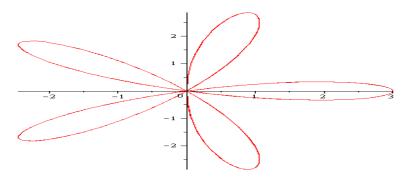
The point $(3; 0)$ belongs to the graph, but the point $(-3; 0)$ does not belong to the graph, so the graph is not symmetric about the origin.

Let point $(x_0; y_0)$ belong to the graph. We check whether point $(-x_0; y_0)$ belongs to the graph. We have

$$\sqrt{x_0^2+y_0^2} = 3\left[-\frac{8x_0^5}{(x_0^2+y_0^2)^{5/2}} - \frac{8y_0^4}{(x_0^2+y_0^2)^2}\cdot\frac{x_0}{\sqrt{x_0^2+y_0^2}} + \frac{10x_0^3}{(x_0^2+y_0^2)^{3/2}} + \frac{6y^2}{x_0^2+y_0^2}\cdot\frac{x_0}{\sqrt{x_0^2+y_0^2}} - \frac{3x_0}{\sqrt{x_0^2+y_0^2}}\right].$$ We did

not obtain the equality (*), therefore generally speaking, the last relation is false. It follows from this that the graph is not symmetric about the y-axis.

Answer. The graph is symmetric about the x-axis but is not symmetric about the y-axis and about the origin.