Answer on Question #53933 – Math – Analytic Geometry

Question

Find the center, vertices, and foci of the ellipse with equation x squared divided by 81 plus y squared divided by 225 equals 1 .

Solution

Definition. The ellipse equation has the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \tag{1}$$

If $b > a$, then b is the semi-major axis and a is the semi-minor axis of the ellipse. The points $A_1(a,0)$,

$A_2(-a,0)$ and $B_1(0,b)$, $B_2(0,-b)$ are the ellipse vertices. The points $F_1(0,c)$ and $F_2(0,-c)$ are the ellipse foci, and $|F_1 F_2|=2c$ is the distance between them. The ellipse parameters a, b and c are related by

$$a^2 = b^2 - c^2. \tag{2}$$

If the ellipse is described by the equation (1), then it is centered at the origin (has center at the point $C(0,0)$).

According to the problem condition, the ellipse equation is

$$\frac{x^2}{81} + \frac{y^2}{225} = 1. \tag{3}$$

Let's rewrite (3) in the form (1):

$$\frac{x^2}{9^2} + \frac{y^2}{15^2} = 1. \tag{3a}$$

As we see from (3a), $a = 9$ and $b = 15$. Using (2) we obtain for the parameter c:

$$c^2 = b^2 - a^2 = 225 - 81 = 144 \Longrightarrow \boxed{c = 12}. \tag{4}$$

Hence, using the definition of the ellipse and (3), we find

1) the point $C(0,0)$ is the center of the ellipse (3);
2) the points $A_1(9,0)$, $A_2(-9,0)$ and $B_1(0,15)$, $B_2(0,-15)$ are the vertices of the ellipse (3);
3) the points $F_1(0,12)$ and $F_2(0,-12)$ are the foci of the ellipse (3).
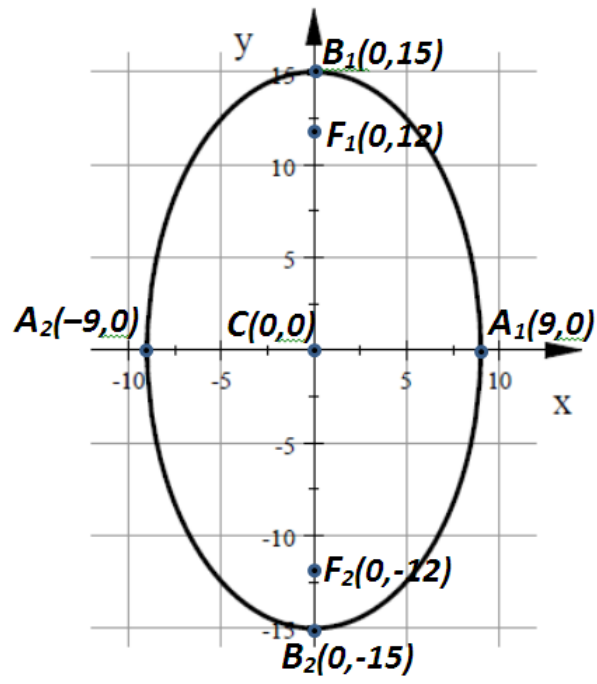
The sketch of the ellipse (3) is shown in Fig. 1.

Fig.1