

Answer on Question #53931 – Math – Calculus

Write the sum using summation notation, assuming the suggested pattern continues.

$$5 - 15 + 45 - 135 + \dots$$

Solution

If the sum has an odd number of terms, then

$$\begin{aligned} \text{Sum} &= \underbrace{5 - 15 + 45 - 135 + \dots}_{2n+1} = 5 \underbrace{(1 - 3^1 + 3^2 - 3^3 + \dots)}_{2n+1} = 5 \underbrace{\left(1 + 3^2 + \dots\right)}_{n+1} - 5 \underbrace{\left(3^1 + 3^3 + \dots\right)}_{n+1} = \sum_{k=0}^{2n} (-1)^k \cdot 5 \cdot 3^k = \\ &= 5 \sum_{k=0}^n 9^k - 15 \sum_{k=1}^n 9^{k-1} = 5 \cdot \left(\frac{1-9^{n+1}}{1-9}\right) - \frac{5}{3} \cdot 9 \left(\frac{1-9^n}{1-9}\right) = \frac{5}{4}(1+3^{2n+1}) \end{aligned}$$

(We used the formula for the sum of a geometric progression).

If the sum has an even number of terms (is is found in a similar way), then

$$\begin{aligned} \text{Sum} &= 5 - 15 + 45 - 135 + \dots = \sum_{k=0}^{2n+1} (-1)^k \cdot 5 \cdot 3^k = 5 \sum_{k=0}^{2n+1} (-1)^k \cdot 3^k = 5 \sum_{k=0}^n 9^k - 15 \sum_{k=1}^{n+1} 9^{k-1} = \\ &= 5 \cdot \left(\frac{1-9^{n+1}}{1-9}\right) - 15 \left(\frac{1-9^{n+1}}{1-9}\right) = \frac{5}{4}(1-9^{n+1}) = \frac{5}{4}(1-3^{2n+2}) \end{aligned}$$

If the series has an odd number of terms, then

$$\sum_{k=0}^{2n} (-1)^k \cdot 5 \cdot 3^k = \frac{5}{4}(1+3^{2n+1}), \text{ let } m = 2n, \text{ then } \sum_{k=0}^{2n} (-1)^k \cdot 5 \cdot 3^k = \frac{5}{4}(1+3^{m+1})$$

If the series has an even number of terms, then

$$\sum_{k=0}^{2n+1} (-1)^k \cdot 5 \cdot 3^k = \frac{5}{4}(1-3^{2n+2}), \text{ let } m = 2n, \text{ then } \sum_{k=0}^{2n+1} (-1)^k \cdot 5 \cdot 3^k = \frac{5}{4}(1-3^{m+1})$$

The general formula for the sum:

$$Sum = \frac{5}{4} \left(1 + (-1)^m 3^{m+1} \right)$$

Let us check

$$m = 0 \Rightarrow Sum = \frac{5}{4} \left(1 + (-1)^0 3^{0+1} \right) = 5 \text{ or } 5$$

$$m = 1 \Rightarrow Sum = \frac{5}{4} \left(1 + (-1)^1 3^{1+1} \right) = \frac{5}{4} (1 - 9) = -10 \text{ or } 5 - 15 = -10$$

$$m = 2 \Rightarrow Sum = \frac{5}{4} \left(1 + (-1)^2 3^{2+1} \right) = \frac{5}{4} (1 + 27) = 35 \text{ or } 5 - 15 + 45 = 35$$

$$m = 3 \Rightarrow Sum = \frac{5}{4} \left(1 + (-1)^3 3^{3+1} \right) = \frac{5}{4} (1 - 81) = -100 \text{ or } 5 - 15 + 45 - 135 = -100$$