Answer on Question #53904 – Math – Algebra

In the binomial expansion of $(p + qx)^r$ where $0 < p < q < r < 8$ the constant is 16 and the coefficients of the terms in x^2 and x^3 are equal. Find the values of the integers p, q and r.

Solution

Constant: $\boldsymbol{p^r = 16}$;

The coefficient of the term in $\boldsymbol{x^2}$**:** $\boldsymbol{C(r,2)q^2p^{r-2} = \frac{r(r-1)}{2}q^2p^{r-2}}$

The coefficient of the term in $\boldsymbol{x^3}$**:** $\boldsymbol{C(r,3)q^3p^{r-3} = \frac{r(r-1)(r-2)}{6}q^3p^{r-3}}$

So, to find p, q and r we have 2 equations:

$\boldsymbol{p^r = 16}$ **and** $\boldsymbol{\frac{r(r-1)}{2}q^2p^{r-2} = \frac{r(r-1)(r-2)}{6}q^3p^{r-3}}$ **or**

$\boldsymbol{p^r = 16}$ **and** $\boldsymbol{3p = (r-2)q}$.

Equation $\boldsymbol{p^r = 16}$ **for** $\boldsymbol{0 < p < r}$ **has only one solution:** $\boldsymbol{p = 2, r = 4}$.

Thus from equation $\boldsymbol{3p = (r-2)q}$ **we have** $\boldsymbol{6 = 2q \rightarrow \ q = 3}$.

And finally: $\boldsymbol{(p + qx)^r = (2 + 3x)^4}$.