Use graphs and tables to find the limit and identify any vertical asymptotes $f(x)$

$$f(x) = \frac{1}{(x-7)^2}$$

Solution

To find the vertical asymptotes, we must set the denominator equal to 0 and solve the following equation:
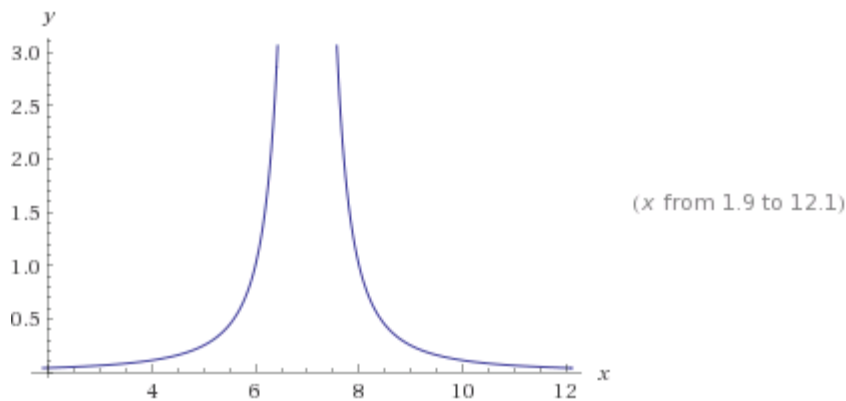
$$(x - 7)^2 = 0$$
$$x = 7$$

Thus, $x = 7$ is the vertical asymptote.

x	6.9	6.99	6.999	6.9999	7.0001	7.001	7.01	7.1
f(x)	100	10000	1000000	100000000	100000000	1000000	10000	100

$$\lim_{x\to 7^-} f(x) = \lim_{x\to 7^-} \frac{1}{(x-7)^2} = \lim_{\varepsilon\to 0,\varepsilon>0} \frac{1}{(7-\varepsilon-7)^2} = \lim_{\varepsilon\to 0,\varepsilon>0} \frac{1}{(-\varepsilon)^2} = \lim_{\varepsilon\to 0,\varepsilon>0} \frac{1}{\varepsilon^2} = +\infty$$

$$\lim_{x\to 7^+} f(x) = \lim_{x\to 7^+} \frac{1}{(x-7)^2} = \lim_{\varepsilon\to 0,\varepsilon>0} \frac{1}{(7+\varepsilon-7)^2} = \lim_{\varepsilon\to 0,\varepsilon>0} \frac{1}{\varepsilon^2} = +\infty$$

$$\lim_{x\to 7} f(x) = \lim_{x\to 7^-} f(x) = \lim_{x\to 7^+} f(x) = +\infty$$

(x from 1.9 to 12.1)

Answer: $\lim_{x\to 7} f(x) = +\infty$, **vertical asymptote is** $x = 7.$