Answer on Question #53836 – Math – Calculus

Find the limit of the function algebraically.

Limit as x approaches zero of quantity nine plus x divided by x to the third power.

Solution

$$\lim_{x\to 0+}\frac{9+x}{x^3}=\lim_{\varepsilon\to 0,\varepsilon>0}\frac{9+(0+\varepsilon)}{(0+\varepsilon)^3}=\lim_{\varepsilon\to 0,\ \varepsilon>0}\frac{9+\varepsilon}{\varepsilon^3}=+\infty.$$

By plugging in ε=0, you get 0 in the denominator. By plugging in ε =0, you get 9 in the numerator. This means that the limit is either positive or negative infinity. If the numerator is positive, then the limit is lim =+ infinity (positive infinity). If the numerator is negative, then the limit is

lim = - infinity (negative infinity).

Therefore our numerator $9+0=9>0$ is positive.

As a result,

$$\lim_{x\to 0+}\frac{9+x}{x^3}=+\infty.$$

Similarly

$$\lim_{x\to 0-}\frac{9+x}{x^3}=\lim_{\varepsilon\to 0,\varepsilon>0}\frac{9-\varepsilon}{(-\varepsilon)^3}=-\lim_{\varepsilon\to 0,\varepsilon>0}\frac{9-\varepsilon}{\varepsilon^3}=-\infty.$$

By the definition of limit, $\lim_{x\to 0}\frac{9+x}{x^3}$ does not exist, because $\lim_{x\to 0-}\frac{9+x}{x^3}\neq\lim_{x\to 0+}\frac{9+x}{x^3}$.