Question

Find the limit of the function by using direct substitution.

limit as x approaches two of quantity x squared plus three x minus one.

Solution

Method 1

(properties of continuous functions)

We shall find the limit of the function by using direct substitution.

Function $f(x) = x^2 + 3x - 1$ is continuous as a polynomial of degree 2, hence

$$\lim_{x \to 2}(x^2 + 3x - 1) = \lim_{x \to 2} f(x) = |definition\ of\ continuous\ function| = f\left(\lim_{x \to 2} x\right) = f(2) =$$

$$= (x^2 + 3x - 1)|_{x=2} = 2^2 + 3 \cdot 2 - 1 = 4 + 6 - 1 = 9, \text{ because } \lim_{x \to 2} x = 2.$$

Method 2

(properties of limits)

Recall the following properties of limits:

$$\lim_{x \to a}\big(g(x) \cdot h(x)\big) = \lim_{x \to a} g(x) \cdot \lim_{x \to a} h(x) \qquad (1)$$

$$\lim_{x \to a}\big(g(x) - h(x)\big) = \lim_{x \to a} g(x) - \lim_{x \to a} h(x) \qquad (2)$$

$$\lim_{x \to a}\big(g(x) + h(x)\big) = \lim_{x \to a} g(x) + \lim_{x \to a} h(x) \qquad (3)$$

$$\lim_{x \to a} c = c, \text{ where } c \text{ is a constant} \qquad (4)$$

$$\lim_{x \to a} x = a \qquad (5)$$

$$\lim_{x \to a} Af(x) = A \lim_{x \to a} f(x) \qquad (6)$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2}(x^2 + 3x - 1) = |apply\ (2), (3)| = \lim_{x \to 2} x^2 + \lim_{x \to 2}(3x) - \lim_{x \to 2} 1 =$$

$$= |x^2 = x \cdot x, apply\ (4)\ to\ the\ third\ term| = \lim_{x \to 2}(x \cdot x) + \lim_{x \to 2}(3x) - 1 =$$

$$= |apply\ (1)\ to\ the\ first\ term| =$$

$$= \lim_{x \to 2} x \cdot \lim_{x \to 2} x + \lim_{x \to 2}(3x) - 1 = |apply\ (5)\ to\ the\ first\ term| = 2 \cdot 2 + \lim_{x \to 2}(3x) - 1 =$$

$$= |apply\ (6)\ to\ the\ second\ term| = 4 + 3 \lim_{x \to 2}(x) - 1 =$$

$$= |apply\ (5)\ to\ the\ first\ term| = 4 + 3 \cdot 2 - 1 = 4 + 6 - 1 = 10 - 1 = 9$$