

Answer on Question #53833 – Math – Calculus

Question

Find the limit of the function by using direct substitution.

limit as x approaches zero of quantity x squared minus eight.

Solution

Method 1

(properties of continuous functions)

We shall find the limit of the function by using direct substitution.

Function $f(x) = x^2 - 8$ is continuous as a polynomial of degree 2, hence

$$\begin{aligned}\lim_{x \rightarrow 0} (x^2 - 8) &= \lim_{x \rightarrow 0} f(x) = |definition\ of\ continuous\ function| = f\left(\lim_{x \rightarrow 0} x\right) = f(0) = \\ &= (x^2 - 8)|_{x=0} = 0^2 - 8 = -8, \text{ because } \lim_{x \rightarrow 0} x = 0.\end{aligned}$$

Method 2

(properties of limits)

Apply the following properties of limits

$$\lim_{x \rightarrow a} (g(x) \cdot h(x)) = \lim_{x \rightarrow a} g(x) \cdot \lim_{x \rightarrow a} h(x) \quad (1)$$

$$\lim_{x \rightarrow a} (g(x) - h(x)) = \lim_{x \rightarrow a} g(x) - \lim_{x \rightarrow a} h(x) \quad (2)$$

$$\lim_{x \rightarrow a} c = c, \text{ where } c \text{ is a constant} \quad (3)$$

$$\lim_{x \rightarrow a} x = a \quad (4)$$

$$\begin{aligned}\lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} (x^2 - 8) = |apply\ (2)| = \lim_{x \rightarrow 0} x^2 - \lim_{x \rightarrow 0} 8 = \\ &= |x^2 = x \cdot x, \text{ apply (3) to the second term}| = \lim_{x \rightarrow 0} (x \cdot x) - 8 = |apply\ (1)| = \\ &= \lim_{x \rightarrow 0} x \cdot \lim_{x \rightarrow 0} x - 8 = |apply\ (4)| = 0 \cdot 0 - 8 = 0 - 8 = -8\end{aligned}$$