Answer on Question #53813 – Math – Statistics and Probability

A machine produces 15% defective components. In a sample of 5, drawn at random use the binomial distribution to find the probabilities that:

A) There'll be 4 defective items

B) There'll not be more than 3 defective items.

C) All the items will be non-defective

Solution

We use the binomial distribution with $p = 0.15, n = 5$.

A) The probability that there'll be 4 defective items is

$$P(k = 4) = \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k} = \frac{5!}{4!\,(5-4)!} 0.15^4 (1-0.15)^{5-4} = 0.00215.$$

B) The probability that there'll not be more than 3 defective items is

$$P(k \le 3) = 1 - P(k > 3) = 1 - \big(P(k = 4) + P(k = 5)\big) =$$
$$= 1 - \left(\frac{5!}{4!\,(5-4)!} 0.15^4 (1-0.15)^{5-4} + \frac{5!}{5!\,(5-5)!} 0.15^5 (1-0.15)^{5-5}\right) =$$
$$= 1 - (0.00215 + 0.00008) = 0.99777.$$

C) The probability that all the items will be non-defective is

$$P(k = 0) = \frac{5!}{0!\,(5-0)!} 0.15^0 (1-0.15)^{5-0} = 0.44371.$$

Answer: A) $\mathbf{0.00215}$; B) $\mathbf{0.99777}$; C) $\mathbf{0.44371}$.