Answer on Question #53801 – Math– Algebra

Question

The cable of a uniformly loaded suspension bridge hang in the form of a parabola .The roadway which is horizontal and 100 meter long is supported by vertical wires attached to the cable, the longest wire being 30 meter and the shortest being 6 meter .Find the length of a supporting wire attached to the roadway 18 meter from the middle

Solution

Suppose that the bridge is a parabolic arc with the vertex *O* (fig.1). The following information is given:
AB = L = 100m is the length of the roadway;
PB = OP/2 = L/2 = 50m;
OP = h=6m is the shortest vertical wire;
SB = H = 30m is the longest vertical wire;
PQ = l = 18m.

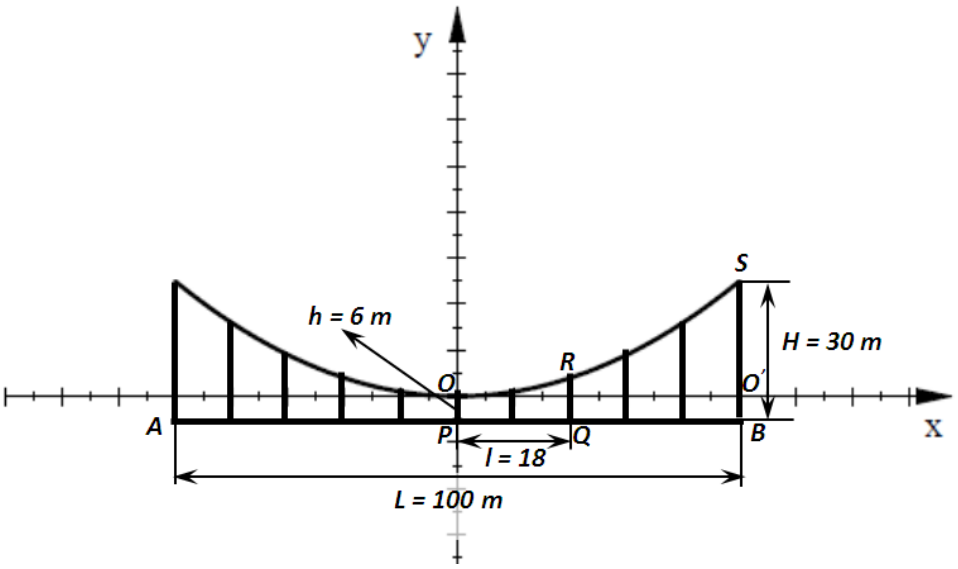

Fig. 1

Let's write coordinates of the points R and S:

$$R(18, y),\ S(50, y').\tag{1}$$

Taking into account that *OP = O'B* , we find the longest vertical wire above x-axis:

$$SO' = SB - O'B = H - h = 30 - 6 = 24m.\tag{2}$$

Hence, from (1), (2) we obtain $SO' = y' = 24m$ and

$$R(18, y),\ S(50,24).\tag{3}$$

The equation of the parabola has the form

$$y = ax^2.\tag{4}$$

Find the parameter *a*. Since the parabola passes through the point $S(50,24)$, (4) gives that

$$S(50,24):\quad 24 = a \cdot (50)^2 \Longrightarrow$$

$$a = \frac{24}{(50)^2} = \frac{24}{2500} = \frac{6}{625}.\tag{5}$$

Further using (4) and (5) the ordinate y of the point $R(18, y)$ on the parabola will be

$$R(18, y): \quad y = a \cdot (18)^2 = \frac{6}{625} \cdot 324 = \frac{1944}{625}. \tag{6}$$

The length of a supporting wire attached to the roadway 18m from the middle is given by

$$RQ = y + h = y + 6. \tag{7}$$

Substituting *y* from (6) into (7) we get the required length:

$$RQ = \frac{1944}{625} + 6 = 3.1104 + 6 \approx 9.11m. \tag{8}$$

Answer: $RQ \approx 9.11m$ is the length of a supporting wire attached to the roadway 18 meter from the middle.