Answer on Question #53689 – Math – Differential Equations

The number of bacteria doubles after every hour. If there were 30 bacteria originally find the number of bacteria after 3 days.

Solution

At first, we must construct the differential equation:

$\frac{dy}{dt} = ky$, where $y = y(t)$ is the number of bacteria at time t, k is the proportionality factor.

We will rewrite equation

$$\frac{dy}{y} = kdt$$

and integrate

$$\int \frac{dy}{y} = k \int dt.$$

After integrating we will have:

$\ln y = kt + \ln C$. So we have

$y(t) = y_0 e^{kt}$, where $y_0 = \ln C = 30$ is the number of bacteria at time t = 0.

After 1 hour the number of bacteria doubles:

$y(1) = 60 = 30e^k$. From this, we have: $e^k = 2$. This gives that

$y(t) = 30 \cdot 2^t$.

From the last formula we can find the number of bacteria after 3 days (3 days equals 72 hours):

$y(72) = 30 \cdot 2^{72} \approx 30 \cdot 4.7 \cdot 10^{21} = 141 \cdot 10^{21}$.

Answer: after 3 days the number of bacteria will be $141 \cdot 10^{21}$.