Answer on Question #53663 – Math – Calculus

Question

Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (If an answer does not exist, enter DNE.) f(t) = 9 cos t, −3π/2 ≤ t ≤ 3π/2

Solution

According to the statement of the problem we have

$$f(t) = 9\cos(t),\ t \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]. \quad (1)$$

Let`s sketch the graph of the given function (See Fig.1)

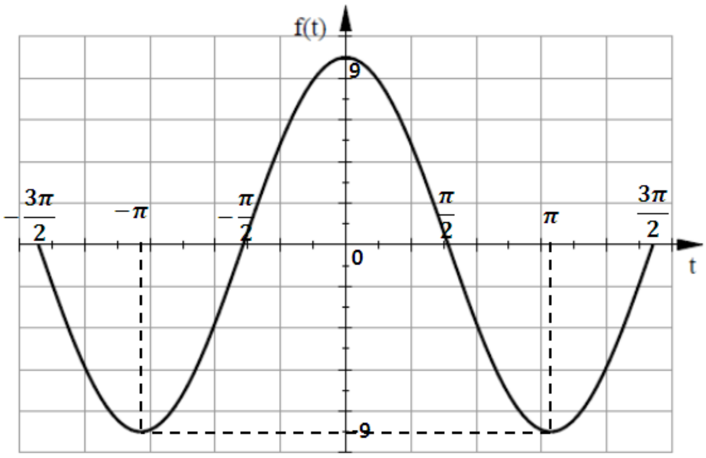

Fig.1

Let’s solve the given problem in a few steps:

1) determination of the critical values;
2) determination of the local min/max values;
3) determination of the absolute min/max values.

1) Find the critical values of *f(t)*.
   As we know, the critical values of *f(t)* are numbers in the domain of *f(t)*, where *f’(t) = 0* or *f’(t)* DNE. So we start by taking the derivative in (1):

$$f'(t) = -9\sin(t).$$

Note that there are no numbers at which $f'(t)$ does not exist, because the sine function is continuous and defined everywhere on the real axis. It follows from equation $f'(t) = 0$ that the critical values of *f(t)* on the interval $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$ are *t = −π, 0, −π*.

2) Find the local max/min values of *f(t)*.

Taking into account (1), evaluate *f(t)* at each endpoint of $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$ and critical value on the interval $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$

$$f\left(-\frac{3\pi}{2}\right)=9\cos\left(-\frac{3\pi}{2}\right)=0 \text{ — there is an endpoint local maximum;}$$
$$f(-\pi)=9\cos(-\pi)=-9 \text{ — there is a local minimum;}$$
$$f(0)=9\cos(0)=9 \text{ — there is a local maximum;}$$
$$f(\pi)=9\cos(\pi)=-9 \text{ — there is a local minimum;}$$
$$f\left(\frac{3\pi}{2}\right)=9\cos\left(\frac{3\pi}{2}\right)=0 \text{ – there is an endpoint local maximum.}$$

3) Find the absolute max/min values of *f(t)* on $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$.

Taking into account (1),

the largest value of function *f(t)* is

$$max_{-\frac{3\pi}{2}\le t\le\frac{3\pi}{2}}f(t)=max\left\{f\left(-\frac{3\pi}{2}\right),f(-\pi),f(0),f(\pi),f\left(\frac{3\pi}{2}\right)\right\}=max\{0,-9,9,-9,0\}=9$$

and the smallest value of function *f(t)* is

$$min_{-\frac{3\pi}{2}\le t\le\frac{3\pi}{2}}f(t)=min\left\{f\left(-\frac{3\pi}{2}\right),f(-\pi),f(0),f(\pi),f\left(\frac{3\pi}{2}\right)\right\}=min\{0,-9,9,-9,0\}=-9.$$

Therefore, the absolute maximum of *f(t)* on $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$ is 9, occurring at *t = 0*, and the absolute minimum of *f(t)* on $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$ is -9, occurring at $t=-\pi,\ \pi$.

Thus, the function $f(t)$ in (1) has both local and absolute maximum of *9* at *t = 0: f(0) = 9*; and it also has both local and absolute minimums of *– 9* at $t=-\pi,\ \pi$: $f(-\pi)=f(\pi)=-9$.

Answer: $(t,f(t))=(0,9)$ is both a local and the absolute maximum of function $f(t)$ on $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$; $(t_1,f(t_1))=(-\pi,-9)$ and $(t_2,f(t_2))=(\pi,-9)$ are both local and absolute minimums of function $f(t)$ on $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$.